ALGEBRA FOR

PREPARATORY THREE

SECOND TERM

Sheet (1)

Solving 2 equations of first degree in 2 variables

First — Solving two equations of the first degree in two variables graphically

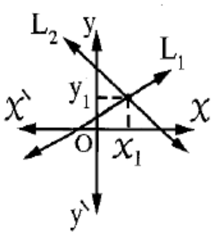
- The meaning of solving two equations graphically is finding the ordered pair or ordered pairs which satisfy the two equations simultaneously.

  Since the set of solution of the equation of the first degree in two variables in $\mathbb{R} \times \mathbb{R}$ is represented graphically by a straight line.

Then to solve the two equations graphically, we do as follows :

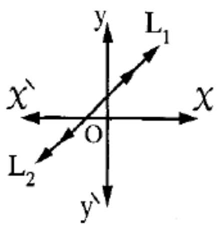
In the Cartesian plane draw the two straight lines which represent the two equations to be L_1 and L_2 , then the S.S. is the point of intersection of the two straight lines L_1 and L_2 , then we have three cases.

1 L_1 and L_2 intersect at the point (x_1 , y_1)



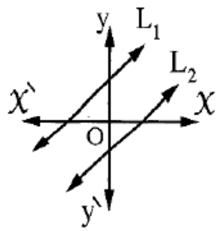
- There is a unique solution (x_1 , y_1)
- The S.S. = $\{(x_1 , y_1)\}$

2 L_1 and L_2 are coincident



- There is an infinite number of solutions

3 L_1 and L_2 are parallel



- There is no solution
- The S.S. = $\varnothing$

The following examples in the following table show each case of the previous cases.

Example (1)

$L_1 : 2x - y = 5$

$L_2 : x + 3y + 1 = 0$

$\because L_1 : y = 2x - 5$

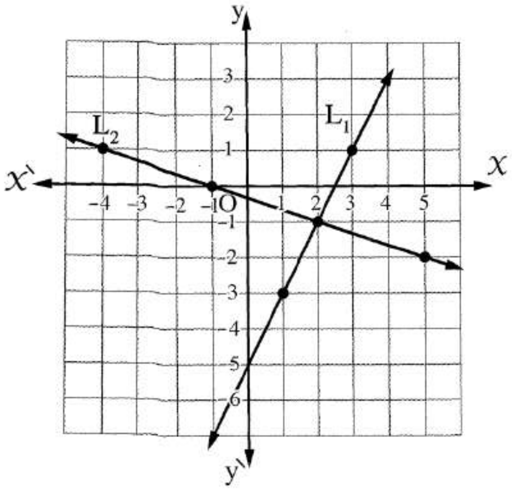
$\therefore$

x	1	2	3
y	-3	-1	1

$\because L_2 : x = -3y - 1$

$\therefore$

x	-1	-4	5
y	0	1	-2



The solution set in $\mathbb{R}^2 = \{(2, -1)\}$

Example (2)

$L_1 : y = 2x - 4$

$L_2 : 4x = 2y + 8$

$\because L_1 : y = 2x - 4$

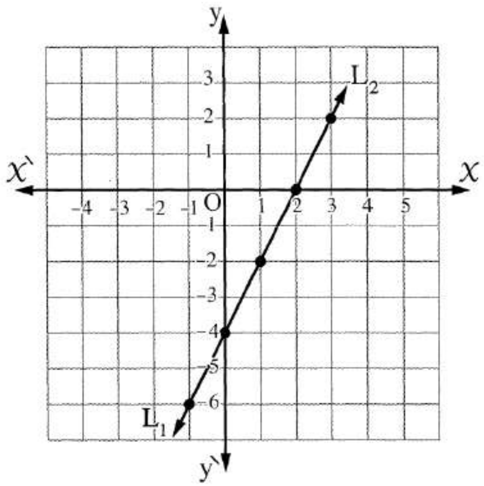
$\therefore$

x	0	1	-1
y	-4	-2	-6

$\because L_2 : x = \frac{2y + 8}{4} = \frac{1}{2} y + 2$

$\therefore$

x	2	3	1
y	0	2	-2



The solution set in $\mathbb{R}^2$

$= \{(x, y) : y = 2x - 4, (x, y) \in \mathbb{R}^2\}$

Example (3)

$L_1 : y = 2x - 2$

$L_2 : 2y - 4x - 2 = 0$

$\because L_1 : y = 2x - 2$

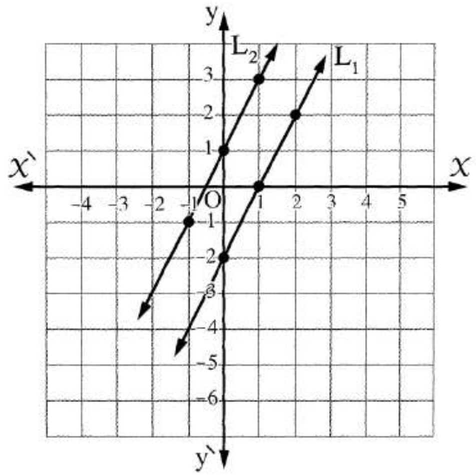
$\therefore$

x	2	1	0
y	2	0	-2

$\because L_2 : y = 2x + 1$

$\therefore$

x	0	1	-1
y	1	3	-1



The solution set in $\mathbb{R}^2 = \varnothing$

Remark

We can recognise the number of solutions of any two equations of the first degree in two variables by knowing the slope of the straight line and determining the point of its intersection with y-axis as follows :

We find the slopes of the two straight lines

If

$m_1 = m_2$

We find the points of intersection of the two straight lines with y-axis

If

The two points are equals

Then the two straight lines are coincident and the number of solutions is an infinite number.

The two points are different

Then the two straight lines are parallel and the number of solutions = 0

$m_1 \neq m_2$

The two straight lines intersect at one point , then we say the number of solutions = 1

Second Solving two equations of the first degree in two variables algebraically

This method depends on removing one of the two variables to get an equation of the first degree in one variable , then we get the value of this variable by solving this equation.

Then we substitute by this value in any of the two equations to get the value of the other variable which we have removed before.

For that purpose , we follow one of the two methods :

1 Substituting method.

2 Omitting method.

Choose the correct answer :

1 The two straight lines : $2x = 3$ and $3y = 5$ are .... *(El-Dakahlia 2013)*

(a) perpendicular. (b) coincident. (c) parallel. (d) intersecting.

2 The S.S. of the two equations : $2x - y = 2$, $x + y = 7$ in $\mathbb{R} \times \mathbb{R}$ is .... *(Kafr El-Sheikh 2014)*

(a) $\{(1 , 0)\}$ (b) $\{(2 , 2)\}$ (c) $\{(3 , 4)\}$ (d) $\{(2 , 5)\}$

3 📖 The S.S. of the two equations : $x - 2y = 1$, $3x + y = 10$ in $\mathbb{R} \times \mathbb{R}$ is ....

(Port Said 2013 , El-Fayoum 2011)

(a) $\{(5, 2)\}$ (b) $\{(2, 4)\}$ (c) $\{(1, 3)\}$ (d) $\{(3, 1)\}$

4 📖 If there are infinite numbers of solutions of the two equations :

$x + 4y = 7$, $3x + ky = 21$, then k = ... *(Cairo 2014 , Qena 2013 , El-Dakahlia 2012)*

(a) 4 (b) 7 (c) 12 (d) 21

Essay problems:

1 **Find graphically in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations :**

$y = 2x - 3$, $x + 2y = 4$ *(Port Said 2014)* « $\{(2, 1)\}$ »

2 **Find graphically in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations :**

$x + y = 5$, $x - y = 1$ *(S. Sinai 2013)* « $\{(3, 2)\}$ »

3 **Find in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations :**

$2x - y = 3$, $x + 2y = 4$ *(Assiut 2013, El-Sharkia 2014)* « $\{(2, -1)\}$ »

4 **Find in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations :**

$3x + 4y = 24$, $x - 2y = -2$ *(Giza 2012)* « $\{(4, 3)\}$ »

5 📖 **Find the solution set for each pair of the following two equations algebraically and graphically :**

$y = x + 4$, $x + y = 4$ *(Alexandria 2013)* « $\{(0, 4)\}$ »

6 📖 **Find the solution set for each pair of the following two equations algebraically and graphically :**

$x - y = 4$, $3x + 2y = 7$ *(Damietta 2013)* « $\{(3, -1)\}$ »

Homework

1 **Find algebraically in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations**

(1) $x + y = 4$, $x - y = 2$ « {(3 , 1)} »

(2) $4x - y = 5$, $2x + y = 7$ « {(2 , 3)} »

(3) $x + 3y = 2$, $3x + 4y = 6$ « {(2 , 0)} »

(4) $5y + x = 2$, $2x - 3y + 9 = 0$ « {(– 3 , 1)} »

(5) $2y - 3x = 7$, $3y + 2x = 4$ « {(– 1 , 2)} »

2 **Find in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations :**

(1) $2x - y = 3$, $x + 2y = 4$ *(Assiut 17 , El-Ismailia 15)* « {(2 , 1)} »

(2) $3x + 4y = 24$, $x - 2y = -2$ *(Giza 12)* « {(4 , 3)} »

(3) $\frac{x}{6} + \frac{y}{3} = \frac{1}{3}$, $\frac{x}{2} + \frac{2y}{3} = 1$ « {(2 , 0)} »

3 **Find the value of a and b in each of the following :**

📖 $a x + b y - 5 = 0$, $3 a x + b y = 17$

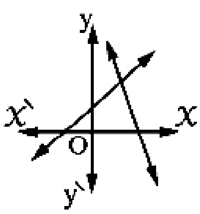
given that (3 , – 1) is a solution for the two equations *(El-Gharbia 2014)* « 2 , 1 »

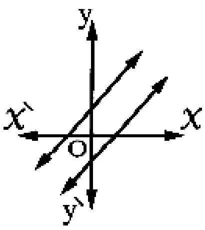
4 If : $f(x) = a x^2 + b$, $f(1) = 5$, $f(2) = 11$, then find the value of a and b « 2 , 3 »

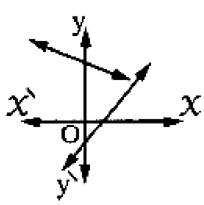
(El-Fayoum 2009)

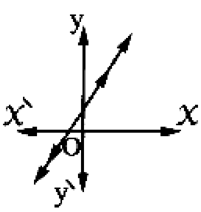
Choose the correct answer :

1 Which of the following graphs represents two equations of the first degree in two variables which have no common solution ?

(a) 

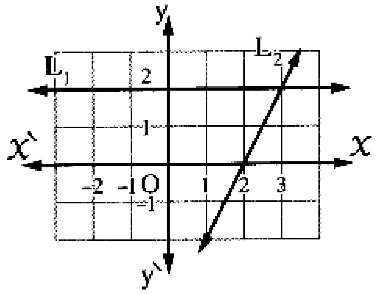
(b) 

(c) 

(d) 

2 In the opposite graph :

The S.S. of the two equations which are represented by the two straight lines L_1 and L_2 is ………



(a) $\{(2 , 2)\}$ (b) $\{(3 , 2)\}$ (c) $\{(2 , 0)\}$ (d) $\{(2 , 3)\}$

3 The point of intersection of the two straight lines : $X + 2 = 0$, $y = X$ is ……… *(El-Dakahlia 17)*

(a) $(2 , 2)$ (b) $(2 , 0)$ (c) $(-2 , -2)$ (d) $(0 , 0)$

4 The two straight lines : $3X = 7$, $2y = 9$ are ……… *(Matrouh 16 , Luxor 16 , El-Sharkia 15)*

(a) parallel. (b) coincident.
(c) intersecting and non perpendicular. (d) perpendicular.

5 The two straight lines : $y = X - 3$, $y = X + 3$ are ………

(a) parallel. (b) perpendicular. (c) coincident. (d) intersecting.

6 The two straight lines representing the two equations : $X + 5y = 1$, $X + 5y - 8 = 0$ are ……… *(El-Beheira 17 , Giza 16)*

(a) parallel. (b) coincident.
(c) perpendicular. (d) intersecting and not perpendlicular.

7 📖 The S.S. of the two equations : $X - 2y = 1$, $3X + y = 10$ in $\mathbb{R} \times \mathbb{R}$ is ……… *(Port Said 13 , El-Fayoum 11)*

(a) $\{(5 , 2)\}$ (b) $\{(2 , 4)\}$ (c) $\{(1 , 3)\}$ (d) $\{(3 , 1)\}$

8 📖 The two straight lines : $3x + 5y = 0$, $5x - 3y = 0$ are intersecting at ....

(Alexandria 14 , El-Beheira 11)

(a) the origin point.
(b) the first quadrant.
(c) the second quadrant.
(d) the fourth quadrant.

9 If the point of intersection of two straight lines : $x - 1 = 0$, $y = 2k$ lies on the fourth quadrant , then k may be equal .... *(Kafr El-Sheikh 16)*

(a) -5
(b) 0
(c) 1
(d) 5

Complete the following:

(1) If $L_1 \cap L_2 = \varnothing$, then the S.S. of the two equations which are represented by the two straight lines L_1 and L_2 are ....

(2) Two equations are represented by the two straight lines L_1 and L_2 and they have an infinite number of solutions , then the two straight lines are ....

(3) The two straight lines which represent the two equations : $x = 3$, $y = 1$ are intersecting at the point ....

(4) The point of intersection of the two straight lines : $x + 3 = 0$, $y - 5 = 0$ lies in the .... quadrant.

(5) 📖 The solution set of the two equations : $x + y = 0$, $y - 5 = 0$ in $\mathbb{R} \times \mathbb{R}$ is ....

(Alex. 11)

(6) 📖 The S.S. of the two equations : $x + 3y = 4$, $3y + x = 1$ in $\mathbb{R} \times \mathbb{R}$ is ....

(7) 📖 The S.S. of the two equations : $4x + y = 6$, $8x + 2y = 12$ in $\mathbb{R} \times \mathbb{R}$ is ....

(8) The unique solution of the two equations : $y = 2$, $2x = y$ in $\mathbb{R} \times \mathbb{R}$ is ....

(9) The S.S. of the two equations : $\frac{x}{2} + 1 = 0$, $y + 5 = 0$ in $\mathbb{R} \times \mathbb{R}$ is .... *(North Sinai 12)*

(10) If $x + y = 5$, $x - y = 3$, then $x^2 - y^2 =$ .... *(Red Sea 11)*

(11) 📖 If the two straight lines which represent the two equations : $x + 3y = 4$, $x + ay = 7$ are parallel , then $a =$ ....

(12) 📖 If there is only one solution for the two equations : $x + 2y = 1$ and $2x + ky = 2$, then k cannot equal ....

Sheet (2)

Applications on solving 2 equations of first degree in 2 variables

In this kind of problems, the solution takes the following steps :

1. Let one of the two unknown be x and the other be y
2. From the given data in the problem, form two equations of the first degree in x and y
3. Solve the two equations algebraically or graphically to get the values of x and y
   It is preferable to solve them algebraically.

Choose the correct answer :

1 The 2-digit number in which the units digit = x and its tens digit = y is ………

(a) $x\,y$ (b) $x + y$ (c) $x + 10\,y$ (d) $10\,x + y$

2 Twice a number formed from two digits if its units digit = x and tens digit = y is ………

(a) $2\,x + 10\,y$ (b) $x + 10\,y$ (c) $2\,x + 2\,y$ (d) $2\,x + 20\,y$

3 Two numbers x and y , x is more than y by 5 , then y = ………

(a) $5\,x$ (b) $x - 5$ (c) $x + 5$ (d) $5 - x$

4 If the number x is more than twice the number y by 3 , then ………

(a) $2\,y - x = 3$ (b) $x + 3 = 2\,y$ (c) $2\,x - y = 3$ (d) $x - 2\,y = 3$

5 If Ahmed's age now is x years and Mohamed's age now is y , then the sum of their ages 5 years ago is ……… years.

(a) $x + y$ (b) $x + y - 5$ (c) $x + y - 10$ (d) $x + y + 10$

Essay problems:

1 The sum of two numbers = 12 and twice one of them is more than the other by 3 Find the two numbers.

2 The sum of two natural numbers is 63 and their difference is 11
Find the two numbers. *(El-Beheira 16)* « 37 , 26 »

3 The sum of two integers is 54 , twice the first number equals the second number.
Find the two numbers. « 18 , 36 »

4 📖 A rectangle is with a length more than its width by 4 cm. If the perimeter of the rectangle is 28 cm. **Find the area of the rectangle.** *(Cairo 17 , Alex. 12)* « 45 cm^2 »

5 If three times a number is added to twice a second number the sum is 13 , and if the first number is added to three times the second number the sum is 16 ,
find the two numbers. *(Port Said 17)* « 1 , 5 »

6 📖 A two-digit number , the sum of its digits is 11 If the two digits are reversed , then the resulted number is 27 more than the original number , what is the original number ?
(Kafr El-Sheikh 16) « 47 »

Homework

Essay problems:

1 📖 If the number of the teams participating in the African Nations Cup is 16 teams , and the number of non-Arab teams is 4 more than three times the Arab teams ,
find the number of the participating Arab teams in the championship. « 3 teams »

2 The sum of ages of a man and his son is 55 years. If the man's age is more than four times his son's age by 5 years. **Find the age of each of them.** « 45 years , 10 years »

3	Two supplementary angles , the twice of the measure of their bigger equals seven times the measure of the smaller. **Find the measure of each angle.** « 140° , 40° »
4	Two acute angles in a right-angled triangle , the difference between their measures is 50° **Find the measure of each angle.** *(Damietta 17 , Kafr El-Sheikh 17 , North Sinai 15)* « 70° , 20° »
5	If the sum of the ages of Ahmed and Osama now is 43 years , and after 5 years the difference between both ages will be 3 years. **Find the age of each of them after 7 years.** « 30 years , 27 years »
6	A two-digit number equals 5 times the sum of its digits. If the two digits are reversed then the resulted number will be more than the origin number by 9 **Find the origin number.** « 45 »

Sheet (3)

Solving an equation of the 2nd degree in one unknown grphically and algebraically

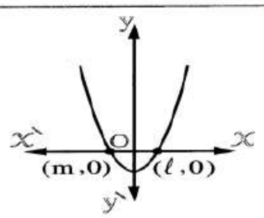
First : Solving an equation of the second degree in one unknown graphically :

To solve an equation of the second degree in one unknown graphically , we do the following steps :

1. Put the equation in the form : $a x^2 + b x + c = 0$
2. Assume that : $f(x) = a x^2 + b x + c$
3. Draw the curve of the function f by the method that you studied previously.
4. Determine the points of intersection of the function curve and x-axis , then the x-coordinates of these points of intersection are the solutions of the equation : $f(x) = 0$
   i.e. $a x^2 + b x + c = 0$

Case (1)

The curve intersects x-axis at **two points**

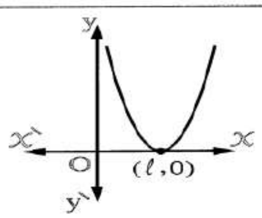


There are **two solutions** in $\mathbb{R}$

The S.S. $= \{\ell , m\}$

Case (2)

The curve **touches** x-axis at **one point**

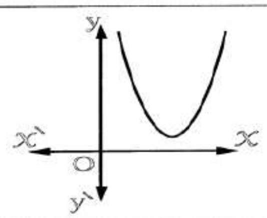


There is **a unique solution** in $\mathbb{R}$

The S.S. $= \{\ell\}$

Case (3)

The curve **does not intersect** x-axis



There is **no solution** in $\mathbb{R}$

The S.S. $= \varnothing$

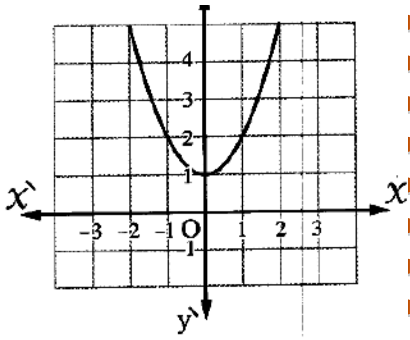
Second : Solving an equation of the second degree in one unknown using the general rule (general formula) :

$$a x^2 + b x + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Choose the correct answer :

1 The opposite figure represents the curve of a quadratic function f , then the solution set of the equation $f(x) = 0$ in $\mathbb{R}$ is .... *(Cairo 16)*

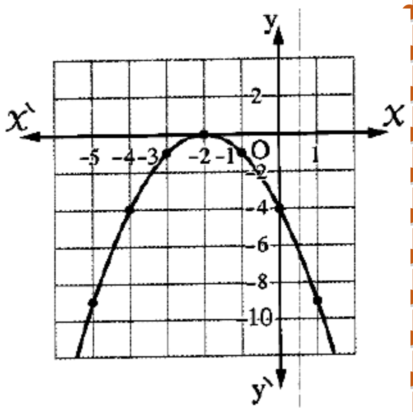


(a) $\varnothing$ (b) $\{1\}$

(c) $\{0\}$ (d) $\{(0 , 1)\}$

2 **In the opposite figure :**

The S.S. of the equation $f(x) = 0$ in $\mathbb{R}$ is ....

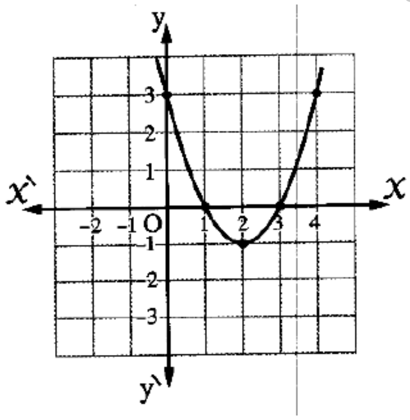


(a) $\{-2\}$ (b) $\{-2 , 4\}$

(c) $\{4\}$ (d) $\varnothing$

3 **In the opposite graph :**

The S.S. of the equation $f(x) = 0$ in $\mathbb{R}$ is .... *(Cairo 15)*



(a) $(2 , -1)$ (b) $\{(3 , 1)\}$

(c) $\{3 , 1\}$ (d) $(3 , 0)$

4 If the curve of the quadratic function does not intersect the X-axis at any point , then the number of solutions of the equation $f(X) = 0$ in $\mathbb{R}$ is .... *(El-Monofia 17 , Qena 04)*

(a) a unique solution.
(b) two solutions.
(c) an infinite number.
(d) zero.

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5 If the curve of the quadratic function f passes through the points $(2, 0)$, $(-3, 0)$ and $(0, -6)$, then the solution set of the equation $f(X) = 0$ in $\mathbb{R}$ is .... *(El- Dakahlia 17)*

(a) $\{-2, 3\}$
(b) $\{3, 2\}$
(c) $\{2, -3\}$
(d) $\{-3, -6\}$

---

6 The curve of the function $f : f(X) = X^2 - 5X$ intersects the X-axis at the two points ....

(a) $(2, 0)$, $(0, 5)$
(b) $(0, 0)$, $(5, 0)$
(c) $(2, 0)$, $(-5, 0)$
(d) $(0, 0)$, $(-5, 0)$

Essay problems:

1 **Find the S.S. of the following equation in $\mathbb{R}$: $X^2 + 2X - 3 = 0$:**

(1) graphically on the interval $[-4, 2]$
(2) using factorization.
(3) using the general formula.
(4) using the calculator.

---

2 Represent graphically the function $f : f(X) = X^2 - 2X$ in the interval $[-1, 3]$, from the graph find the S.S. of the equation : $X^2 - 2X = 0$ *(Suez 12)*

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3 **Find in $\mathbb{R}$ the S.S. of each of the following equations using the general formula :**

(1) $X^2 + 7X + 2 = 0$ approximating the result to the nearest tenth. *(El-Kalyoubia 16)*

(2) 🕮 $X^2 - 4X + 1 = 0$ approximating the result to the nearest two decimal digits. *(Giza 17 , Aswan 14 , Alexandria 13)*

(3) 🕮 $2X^2 - 4X + 1 = 0$ rounding the result to three decimal digits. *(Qena 12)*

(4) 🕮 $3X^2 - 6X + 1 = 0$ rounding the result to the nearest three decimals. *(South Sinai 15)*

4 When a dolphin jumps over water surface , its pathway follows the relation $y = -0.2\,x^2 + 2\,x$ where y is the height of the dolphin above water surface and x is the horizontal distance in feet.

Find the horizontal distance that the dolphin covers when it jumps from water till it returns again to water. « 10 feet »

Homework

Choose the correct answer :

1 If the S.S. of the equation : $4\,x^2 + 4\,x + k = 0$ is $\{-\frac{1}{2}\}$, then k = ...

(a) 2 (b) 1 (c) – 1 (d) – 8

2 If $x = 3$ is one of the solutions for the equation : $x^2 - a\,x - 6 = 0$, then a = ...

(Suez 17)

(a) 3 (b) 2 (c) 1 (d) – 1

3 In the equation : $a\,x^2 + b\,x + c = 0$, if $b^2 - 4\,a\,c > 0$, then this equation has .... roots in $\mathbb{R}$

(Damietta 16)

(a) 1 (b) 2 (c) zero (d) ∞

4 In the equation : $a\,x^2 + b\,x + c = 0$, if $b^2 - 4\,a\,c = 0$, then the number of real solutions of the equation = ...

(a) 1 (b) 2 (c) zero (d) an infinite number

5 In the equation : $a\,x^2 + b\,x + c = 0$, if $b^2 - 4\,a\,c < 0$, then the number of roots of the equation in $\mathbb{R}$ = ...

(a) 1 (b) 2 (c) zero (d) an infinite number

6 If $x \in \mathbb{R}$, then the equation : $x^2 + x + 1 = 0$ ....

(a) has two roots. (b) has one root.

(c) has no roots. (d) has an infinite number of roots.

Essay problems:

1 **Graph the function $f : f(x) = x^2 + 2x + 1$ in the interval $[-4, 2]$ and from the graph , find the solution set of the equation : $x^2 + 2x + 1 = 0$**

2 **Draw a graphical representation of the function f where $f(x) = 6x - x^2 - 9$ in the interval $[0, 5]$ and from the drawing find :**

(1) The maximum value or the minimum value of the function.

(2) The solution set of the equation : $6x - x^2 - 9 = 0$ *(Port Said 12)*

3 **Find in $\mathbb{R}$ the solution set of each of the following equations using the general formula approximating the result to three decimal digits :**

(1) $x^2 = 6x - 7$

(2) $2x^2 - 10x = 1$ *(Damietta 13)*

(3) $x(x-1) = 4$ *(Kafr El-Sheikh 16)*

(4) $2x^2 = 3(2 - x)$

(5) $x^2 - 2x + 4 = x + 3$

(6) $(x-3)^2 - 5x = 0$

4 A snake saw a hawk at a height of 160 metres and hawk was flying at a speed of 24 metre / minute to pounce on it. If the hawk is launching vertically downwards according to the relation $d = Vt + 4.9t^2$ where d is the distance by metre , V is the launching speed in metre / minute and t is the time in minutes.

Find the time the snake takes to escape before the hawk reaches it. « less than 3.77 seconds »

Sheet (4)

Solving 2 equations in 2 variables

one of them of 1st degree and the other is of 2nd degree

The method of solving two equations in two variables , one of them is of first degree and the other is of second degree , depends on the substituting method.

Choose the correct answer :

1 The S.S. of the two equations : $x y = 5$, $x + x y = 6$ in $\mathbb{R} \times \mathbb{R}$ is ....

(a) $\{(1 , 5)\}$ (b) $\{(5 , 6)\}$

(c) $\{(5 , 2)\}$ (d) $\{(1 , 5) , (5 , 1)\}$

2 If $x^2 - y^2 = 15$, $x - y = 3$, then $x + y =$ .... *(Cairo 16)*

(a) -5 (b) -3 (c) 3 (d) 5

3 The S.S. of the two equations : $x - y = 0$, $x y = 9$ in $\mathbb{R} \times \mathbb{R}$ is .... *(El-Gharbia 11)*

(a) $\{(0 , 0)\}$ (b) $\{(-3 , 3)\}$

(c) $\{(3 , 3)\}$ (d) $\{(-3 , -3) , (3 , 3)\}$

4 The S.S. of the two equations : $x - 1 = 0$, $x^2 + y^2 = 2$ in $\mathbb{R} \times \mathbb{R}$ is ....

(a) $\{(1 , 1)\}$ (b) $\{(1 , -1)\}$ (c) $\{(1 , -1) , (1 , 1)\}$ (d) $\varnothing$

5 The S.S. of the two equations : $x = 1$, $x^2 - y^2 = 10$ in $\mathbb{R} \times \mathbb{R}$ is ....

(a) $\{(1 , 3)\}$ (b) $\{(1 , -3)\}$ (c) $\{(1 , 3) , (1 , -3)\}$ (d) $\varnothing$

Essay problems:

Find in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations :

1 $x = y$, $x^2 + y^2 = 2$ *(Souhag 09)* « $\{(1 , 1) , (-1 , -1)\}$ »

2	$x - 2y = 0$,	$x^2 - y^2 = 3$	*(Port Said 17)* « $\{(2, 1), (-2, -1)\}$ »
3	$x - y = 0$,	$x^2 + xy + y^2 = 27$	*(Cairo 17)* « $\{(3, 3), (-3, -3)\}$ »
4	$y - 2x = 0$,	$xy = 18$	*(El-Sharkia 14)* « $\{(3, 6), (-3, -6)\}$ »
5	$y = x - 1$,	$y^2 + x = 7$	*(Qena 09)* « $\{(3, 2), (-2, -3)\}$ »
6	$x - y = 1$,	$x^2 + y^2 = 25$	*(El-Beheira 17)* « $\{(-3, -4), (4, 3)\}$ »
7	$x + y = 7$,	$xy = 12$	*(Qena 17)* « $\{(4, 3), (3, 4)\}$ »
8	$y - x = 2$,	$x^2 + xy - 4 = 0$	*(El-Gharbia 17)* « $\{(-2, 0), (1, 3)\}$ »
9	$x - 2y - 1 = 0$,	$x^2 - xy = 0$	« $\{(0, -\frac{1}{2}), (-1, -1)\}$ »
10	$x - y = 10$,	$x^2 - 4xy + y^2 = 52$	« $\{(-2, -12), (12, 2)\}$ »

11 The sum of two real numbers is 9 and the difference between their squares equals 45

Find the two numbers. *(Kafr El-Sheikh 13)* « 7 , 2 »

12 The perimeter of a rectangle is 18 and its area is 18 cm^2.

Find its two dimensions. *(New Valley 16)* « 6 cm. , 3 cm. »

13 📖 A length of a rectangle is 3 cm. more than its width and its area is 28 cm^2.

Find its perimeter. *(El-Fayoum 12)* « 22 cm. »

Homework

Choose the correct answer :

1 The S.S. of the two equations : $x + y = 0$, $x^2 + y^2 = 2$ in $\mathbb{R} \times \mathbb{R}$ is .... *(Assiut 13)*

(a) $\{(0 , 0)\}$ (b) $\{(1 , -1)\}$

(c) $\{(-1 , 1)\}$ (d) $\{(1 , -1) , (-1 , 1)\}$

2 The ordered pair which satisfies each of the two equations : $x y = 2$, $x - y = 1$ is .... *(El-Sharkia 12)*

(a) $(1 , 1)$ (b) $(2 , 1)$ (c) $(1 , 2)$ (d) $\left(\frac{1}{2} , 1\right)$

3 One of the solutions for the two equations : $x - y = 2$, $x^2 + y^2 = 20$ is .... *(Qena 17 , Port Said 14)*

(a) $(-4 , 2)$ (b) $(2 , -4)$ (c) $(3 , 1)$ (d) $(4 , 2)$

4 If $y = 1 - x$, $(x + y)^2 + y = 5$, then $y =$ .... *(El-Fayoum 12)*

(a) 5 (b) 3 (c) -4 (d) 4

5 If $x^2 + x y = 15$, $x + y = 5$, then $x =$ .... *(Cairo 06)*

(a) 3 (b) 4 (c) 5 (d) 6

Essay problems:

1 $y + 2x = 0$, $6x^2 - y^2 = 72$ « $\{(6 , -12) , (-6 , 12)\}$ »

2 $x + y = 0$, $y^2 = x$ *(6^{th} October 11)* « $\{(0 , 0) , (1 , -1)\}$ »

3 $y - x = 3$, $x^2 - 2x + 3y = 15$ *(Alex. 11)* « $\{(-3 , 0) , (2 , 5)\}$ »

4 $x = 0$, $x^2 + y^2 + 4x + 3y - 10 = 0$ *(Ismailia 03)* « $\{(0 , 2) , (0 , -5)\}$ »

5 The sum of two real positive numbers is 17 and their product is 72

Find the two numbers. *(Alex. 09)* « 8 , 9 »

6 A right-angled triangle of hypotenuse length 13 cm. and its perimeter is 30 cm.

Find the lengths of the other two sides. *(El-Monofia 15)* « 5 cm. , 12 cm. »

7 The length of a rectangle is X cm. and its width is y cm. and its area = 77 cm^2

If its length decreases by 2 cm. and its width increases 2 cm.

, then it will become a square.

Find the area of the square. *(North Sinai 05)* « 81 cm^2 »

Sheet (5)

Set of zeros of a polynomial function.

Generally

If f is a polynomial function in X , then the set of values of X which makes $f(X) = 0$ is called the set of zeroes of the function f and is denoted by $z(f)$

i.e. $z(f)$ is the solution set of the equation $f(X) = 0$ in $\mathbb{R}$

Notice the difference among f , $f(X)$, $z(f)$:

- f denotes to the function
- $f(X)$ denotes to the rule of the function or the image of X by the function f
- $z(f)$ denotes to the set of zeroes of the function f and it is the solution set of the equation $f(X) = 0$ in $\mathbb{R}$

Choose the correct answer :

1 The set of zeroes of the function $f : f(X) = -3X$ is ....

(Alexandria 2014 , El-Fayoum 2013)

(a) $\{0\}$ (b) $\{-3\}$ (c) $\{-3 , 0\}$ (d) $\mathbb{R}$

2 The set of zeroes of the function $f : f(X) = 5$ is .... *(Alex. 2005)*

(a) $\{5\}$ (b) $\{0\}$ (c) $\mathbb{R}$ (d) $\varnothing$

3 The set of zeroes of the function $f : f(X) = \text{zero}$ is .... *(Qena 2009)*

(a) $\varnothing$ (b) $\mathbb{R} - \{0\}$ (c) $\mathbb{R}$ (d) zero

4 The set of zeroes of the function $f : f(X) = X^2 - 25$ is .... *(Southern Sinai 2014)*

(a) $\{0\}$ (b) $\{5\}$ (c) $\{-5\}$ (d) $\{-5, 5\}$

5 The set of zeroes of the function $f : f(X) = X^4 + X$ is .... *(New Valley 2008)*

(a) $\varnothing$ (b) $\{0\}$ (c) $\{0, 1\}$ (d) $\{0, -1\}$

6 The set of zeroes of the function $f : f(X) = X^6 - 32X$ is .... *(Beni Suef 2011)*

(a) $\{0, 2\}$ (b) $\{2, 16\}$ (c) $\{6, 16\}$ (d) $\{0, 5\}$

7 If $f(X) = X^2 + X + 1$, then the set of zeroes of the function f is .... *(Fayoum 2006)*

(a) $\{0\}$ (b) $\{1\}$ (c) $\varnothing$ (d) $\{2\}$

8 📖 The set of zeroes of the function $f : f(X) = X(X^2 - 2X + 1)$ is ....

(Alexandria 2013)

(a) $\{0, 1\}$ (b) $\{0, -1\}$ (c) $\{0\}$ (d) $\{1\}$

9 📖 If $z(f) = \{2\}$, $f(X) = X^3 - m$, then $m =$ .... *(El-Sharkia 2014, El-Dakahlia 2013)*

(a) $\sqrt[3]{2}$ (b) 2 (c) 4 (d) 8

10 📖 If $z(f) = \{5\}$, $f(X) = X^3 - 3X^2 + a$, then $a =$ .... *(Port Said 2014, Assiut 2011)*

(a) -50 (b) -5 (c) 5 (d) 50

11 If $\{2\}$ is the set of zeroes of the function $f : f(X) = X^2 - 2aX + a^2$, then $a =$ ....

(a) 2 (b) -2 (c) 4 (d) -4

(El Wadi El-Gedied 2014)

Essay problems:

Determine the set of zeroes of the polynomial functions which are defined by the following rules in $\mathbb{R}$:

1 $f(X) = 5X + 10$

2 $f(X) = X^2 - 2X$

3 $f(x) = x^2 - 16$

4 $f(x) = x^2 + 9$

5 $f(x) = x^3 - 125$

6 $f(x) = 5x^3 - 20x$

7 $f(x) = 2x^4 + 54x$

8 $f(x) = 2x^2 - 5x - 12$

9 $f(x) = x(x - 5) - 14$

10 $f(x) = 2x^4 + x^3 - 6x^2$

11 If the function $f : f(x) = x^3 - 2x^2 - 75$

Prove that : The number 5 is the one of the zeroes of the function f *(Beni Suef 15)*

12 If the set of zeroes of the function : $f(x) = a x^2 + x + b$ is $\{0, 1\}$

Find the value of each two constants a and b *(Alex. 17)* « -1, 0 »

13 If the set of zeroes of the function f where $f(x) = a x^2 + b x + 15$ is $\{3, 5\}$

Find the values of a and b « 1, -8 »

Homework

Choose the correct answer :

1 The set of zeroes of the function $f : f(x) = -3x$ is ....

(Alexandria 2014 , El-Fayoum 2013)

(a) $\{0\}$ (b) $\{-3\}$ (c) $\{-3, 0\}$ (d) $\mathbb{R}$

2 The set of zeroes of the function $f : f(x) = x^4 + x$ is .... *(New Valley 2008)*

(a) $\varnothing$ (b) $\{0\}$ (c) $\{0, 1\}$ (d) $\{0, -1\}$

3 If $z(f) = \{2\}$, $f(x) = x^3 - m$, then m = ... *(El-Sharkia 2014 , El-Dakahlia 2013)*

(a) $\sqrt[3]{2}$ (b) 2 (c) 4 (d) 8

4 The set of zeroes of the function $f : f(x) = 5$ is ……… (Alex. 2005)

(a) $\{5\}$ (b) $\{0\}$ (c) $\mathbb{R}$ (d) $\varnothing$

5 The set of zeroes of the function $f : f(x) = x^6 - 32x$ is ……… (Beni Suef 2011)

(a) $\{0, 2\}$ (b) $\{2, 16\}$ (c) $\{6, 16\}$ (d) $\{0, 5\}$

6 If $z(f) = \{5\}$, $f(x) = x^3 - 3x^2 + a$, then $a =$ ……… (Port Said 2014, Assiut 2011)

(a) -50 (b) -5 (c) 5 (d) 50

7 The set of zeroes of the function $f : f(x) =$ zero is ……… (Qena 2009)

(a) $\varnothing$ (b) $\mathbb{R} - \{0\}$ (c) $\mathbb{R}$ (d) zero

8 If $f(x) = x^2 + x + 1$, then the set of zeroes of the function f is ……… (Fayoum 2006)

(a) $\{0\}$ (b) $\{1\}$ (c) $\varnothing$ (d) $\{2\}$

9 If $\{2\}$ is the set of zeroes of the function $f : f(x) = x^2 - 2ax + a^2$, then $a =$ ………

(a) 2 (b) -2 (c) 4 (d) -4

(El Wadi El-Gedied 2014)

10 The set of zeroes of the function $f : f(x) = x^2 - 25$ is ……… (Southern Sinai 2014)

(a) $\{0\}$ (b) $\{5\}$ (c) $\{-5\}$ (d) $\{-5, 5\}$

11 The set of zeroes of the function $f : f(x) = x(x^2 - 2x + 1)$ is ………

(Alexandria 2013)

(a) $\{0, 1\}$ (b) $\{0, -1\}$ (c) $\{0\}$ (d) $\{1\}$

Essay problems:

Determine the set of zeroes of the polynomial functions which are defined by the following rules in $\mathbb{R}$:

1 $f(x) = 6x^2 - 2x^3 - 4x$

2 $f(x) = 25 - 9x^2$

3 $f(x) = x^2 - 3x - 4$

4 $f(x) = x^2 + 2x - 6$

5 | $f(x) = (x-2)(x+3)+4$ *(El-Monofia 15)*

Sheet (6)

Algebraic fractional function.

Definition

If p and k are two polynomial functions , z (k) is the set of zeroes of the function k ,

then the function n where $n : \mathbb{R} - z(k) \longrightarrow \mathbb{R}$, $n(x) = \frac{p(x)}{k(x)}$

n is called a real algebraic fractional function or briefly it is called an algebraic fraction.

Remark

The set of zeroes of the algebraic fractional function is the set of values which makes its numerator equals zero and its denominator does not equal zero.

i.e. The set of zeroes of the algebraic fractional function

= the set of zeroes of the numerator – the set of zeroes of the denominator.

For example:

- If the function $n : n(x) = \frac{x^2+3x}{x^2-9}$, then $n(x) = \frac{x(x+3)}{(x-3)(x+3)}$

  i.e. $z(n) = \{0, -3\} - \{3, -3\} = \{0\}$

- If the function $n : n(x) = \frac{3x+6}{x^2+x-2}$, then $n(x) = \frac{3(x+2)}{(x-1)(x+2)}$

  i.e. $z(n) = \{-2\} - \{1, -2\} = \varnothing$

The common domain of two algebraic fractions or more

- The common domain of two algebraic fractions is the set of real numbers that makes the two algebraic fractions identified together (at the same time)
- **Assume that we have the two algebraic fractions n_1 and n_2 where :**

$n_1(x) = \frac{3}{x-2}$ and $n_2(x) = \frac{5x}{x^2-1}$,

then the domain of n_1 (say) $m_1 = \mathbb{R} - \{2\}$ (because n_1 is undefined when $x = 2$)

and the domain of n_2 (say) $m_2 = \mathbb{R} - \{1, -1\}$ (because n_2 is undefined when $x = 1$ or $x = -1$)

According to that :

$= \mathbb{R}$ – the set of zeroes of the two denominators

(because n_1 and n_2 are undefined together when $x = 2$ or $x = 1$ or $x = -1$)

Choose the correct answer :

1 The domain of the function n : n $(x) = \frac{2x-1}{x^2+1}$ is .... *(North Sinai 2013)*

(a) $\mathbb{R}$ (b) $\mathbb{R}-\{-1\}$ (c) $\mathbb{R}-\{-\frac{1}{2}\}$ (d) $\mathbb{R}-\{\frac{1}{2}\}$

2 The domain of the algebraic fraction $\frac{x-5}{3}$ equals the domain of the algebraic fraction .... *(El-Kalyoubia 16)*

(a) $\frac{x}{x^2+1}$ (b) $\frac{x}{x-3}$ (c) $\frac{3}{x-5}$ (d) $\frac{x-5}{x-3}$

3 If $f(x) = \frac{x}{x-2}$, then $f(2) =$ .... *(Qena 2006)*

(a) 2 (b) 1 (c) zero (d) undefined.

4 If the domain of the function p : p $(x) = \frac{3x}{x^2-4x+\ell}$ is $\mathbb{R}-\{2\}$, then the value of $\ell =$ .... *(Port Said 2003)*

(a) 4 (b) 2 (c) -2 (d) -4

5 The domain of the function $f : f(x) = \frac{2x-4}{x^3-4x}$ is ....

(a) $\mathbb{R}$ (b) $\{-2, 2\}$ (c) $\mathbb{R}-\{-2, 2\}$ (d) $\mathbb{R}-\{-2, 0, 2\}$

6 The common domain of the two fractions $\frac{7}{x-5}$, $\frac{9}{2x-10}$ is .... *(El-Menia 14)*

(a) $\mathbb{R}$ (b) $\mathbb{R}-\{5\}$ (c) $\mathbb{R}-\{2\}$ (d) $\mathbb{R}-\{2, 5\}$

7 The common domain of the two functions $n_1 : n_1(x) = 3x - 15$, $n_2 : n_2(x) = x^2 - 4$ is ....

(a) $\mathbb{R}-\{5\}$ (b) $\mathbb{R}-\{2, -2\}$ (c) $\mathbb{R}-\{5, 2, -2\}$ (d) $\mathbb{R}$

8 If the domain of the function n : n $(x) = \frac{x-2}{x^2+a}$ is $\mathbb{R}$, then a .... 0 *(El-Dakahlia 16)*

(a) = (b) > (c) ≤ (d) <

Essay problems:

1 Determine the domain of each of the algebraic fractional functions which are defined by the following rules :

(1) $n(x) = \frac{x+1}{x-2}$

(2) 🕮 $n(x) = \frac{1}{x+2}$

(3) 🕮 $n(x) = \frac{x+3}{4}$

(4) $n(x) = \frac{x-6}{x}$

(5) 🕮 $n(x) = \frac{x-2}{2x}$

(6) 🕮 $n(x) = \frac{x^2+1}{x^2-x}$

(7) 🕮 $n(x) = \frac{x^2+9}{x^2-16}$

(8) 🕮 $n(x) = \frac{x^2-1}{x^2+1}$

(9) $n(x) = \frac{x^2+25}{x^3+25x}$

(10) $n(x) = \frac{x^2-4}{x^2-x-6}$

(11) $n(x) = \frac{x^2-4x+3}{8x^3+8}$

(12) $n(x) = \frac{x^2-5x+6}{x^4-81}$

2 🕮 If the domain of the function f where $f(x) = \frac{x+b}{x+a}$ is $\mathbb{R} - \{-2\}$ and $f(0) = 3$

, then find the value of each a and b *(El-Fayoum 16)* « 2 , 6 »

3 If the set of zeroes of the function f where $f(x) = \frac{a x^2 - 6x + 8}{bx - 4}$ is $\{4\}$

and its domain is $\mathbb{R} - \{2\}$, then find a , b *(El-Sharkia 17)* « 1 , 2 »

Homework

Choose the correct answer :

1 The domain of the algebraic fraction $\frac{x-5}{3}$ equals the domain of the algebraic fraction .... *(Kafr El-Sheikh 2014)*

(a) $\frac{x}{x^2+1}$ (b) $\frac{x}{x-3}$ (c) $\frac{3}{x-5}$ (d) $\frac{x-5}{x-3}$

2 If the domain of the function p : $p(x) = \frac{3x}{x^2-4x+\ell}$ is $\mathbb{R} - \{2\}$

, then the value of ℓ = ... *(Port Said 2003)*

(a) 4 (b) 2 (c) – 2 (d) – 4

3 The domain of the function n : n $(x) = \frac{2x-1}{x^2+1}$ is ……… (North Sinai 2013)

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{-1\}$ (c) $\mathbb{R} - \{-\frac{1}{2}\}$ (d) $\mathbb{R} - \{\frac{1}{2}\}$

4 If $f(x) = \frac{7+x}{7-x}$, $x \in \mathbb{R} - \{7, -7\}$, then $f(-2) =$ ……… (El-Dakahlia 16)

(a) $\frac{-1}{f(-2)}$ (b) $\frac{-1}{f(2)}$ (c) $\frac{1}{f(2)}$ (d) $\frac{1}{f(-2)}$

Essay problems:

1 **Find the common domain of the following algebraic fractions :**

(1) $\frac{x}{3}$, $\frac{3}{x}$

(2) 📖 $\frac{x+2}{x+5}$, $\frac{x-4}{x-7}$

(3) $\frac{3x}{x-2}$, $\frac{x+3}{x^2-9}$ (North Sinai 09)

(4) $\frac{x^2+x+1}{2x}$, $\frac{x^2-1}{x^2-x}$ (Port Said 03)

(5) 📖 $\frac{x}{x^2-4}$, $\frac{3}{2-x}$

(6) $\frac{x^2+3x}{x^3-9x}$, $\frac{x^2+3x+9}{x^3-27}$

(7) $\frac{x-4}{x^2-5x+6}$, $\frac{2x}{x^3-9x}$

(8) 📖 $\frac{x^2+4}{x^2-4}$, $\frac{7}{x^2+4x+4}$

2 Determine the domain of the function n : n $(x) = \frac{2x+1}{x^2-5x+6}$

, then find n (0) , n (2) (New Valley 08)

3 📖 If the domain of the function n : n $(x) = \frac{x-1}{x^2-ax+9}$ is $\mathbb{R} - \{3\}$

, then find the value of a (Beni Suef 17) « 6 »

4 If n is an algebraic fraction where n $(x) = \frac{11}{4x^2-12x+9}$ and n (a) is undefined

, then find the value of a « $\frac{3}{2}$ »

Sheet (7)

Equality of two algebraic functions.

Reducing the algebraic fraction

Definition

It is said that the algebraic fraction is in its simplest form if there are no common factors between its numerator and denominator.

From the previous , to reduce the algebraic fraction , we do as follows :

1. Factorize each of the numerator and denominator perfectly.
2. Identify the domain of the algebraic fraction before removing the common factors between the numerator and denominator.
3. Remove the common factors between the numerator and denominator to get the simplest form of the algebraic fraction.

Equality of two algebraic fractions

It is said that the two algebraic fractions n_1 and n_2 are equal (i.e. $n_1 = n_2$) if the two following conditions are satisfied together :

1. The domain of n_1 = the domain of n_2
2. $n_1(X) = n_2(X)$ for each $X \in$ the common domain.

Choose the correct answer :

1 If the domain of $n_1 : n_1(X) = \frac{5}{X-8}$ equals the domain of $n_2 : n_2(X) = \frac{X-3}{X+k}$, then k = ...

(a) 8 (b) – 8 (c) – 3 (d) 24

2 If $n_1(X) = \frac{X^2-4}{X-2}$, $n_2(X) = X+2$, then $n_1 = n_2$ when they have the same domain which is .... *(Fayoum 03)*

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{2\}$ (c) $\mathbb{R} - \{-2\}$ (d) $\mathbb{R} - \{1\}$

3 If $n_1(X) = \frac{1}{X-3}$, $n_2(X) = \frac{1}{3-X}$, then $n_1 \neq n_2$ because .... *(Souhag 04)*

(a) $n_1(X) = n_2(X)$

(b) the domain of n_1 = the domain of n_2

(c) $n_1(X) \neq n_2(X)$

(d) the domain of $n_1 \neq$ the domain of n_2

Essay problems:

1 **Reduce each of the following algebraic fractions to the simplest form showing the domain of each of them :**

(1) $n(x) = \dfrac{2x+8}{x+4}$

(2) $n(x) = \dfrac{x^2-2x}{x^2+3x}$

(3) $n(x) = \dfrac{x^2-4x}{x^2-16}$

(4) 📖 $n(x) = \dfrac{x^2-4}{x^3-8}$

(5) $n(x) = \dfrac{12x^2-8x}{6x^2-4x}$

(6) 📖 $n(x) = \dfrac{x^2-4}{x^2-5x+6}$

(7) 📖 $n(x) = \dfrac{x^2-6x+9}{2x^3-18x}$

(8) $n(x) = \dfrac{x^2+x-6}{x^2-2x-15}$

(9) 📖 $n(x) = \dfrac{2x^2+7x+6}{4x^2+4x-3}$

(10) 📖 $n(x) = \dfrac{x^3+1}{x^3-x^2+x}$

2 **In each of the following , prove that : $n_1(x)$ and $n_2(x)$ are equal for all values of x which belong to the common domain and find this domain. (In another meaning , find the common domain in which the two functions n_1 and n_2 are equal) :**

(1) $n_1(x) = \dfrac{4x^2-9}{6x-9}$, $n_2(x) = \dfrac{2x^2+3x}{3x}$ *(Port Said 2015)*

(2) $n_1(x) = \dfrac{x^2-x-2}{x^2+2x+1}$, $n_2(x) = \dfrac{x^2-3x+2}{x^2-1}$

(3) $n_1(x) = \dfrac{x^2-3x+9}{x^3+27}$, $n_2(x) = \dfrac{2}{2x+6}$ *(El-Sharkia 17)*

(4) $n_1(x) = \dfrac{x^2-4}{x^2+x-6}$, $n_2(x) = \dfrac{x^3-x^2-6x}{x^3-9x}$ *(El-Monofia 17)*

(5) $n_1(x) = \dfrac{x^2+x-12}{x^2+5x+4}$, $n_2(x) = \dfrac{x^2-2x-3}{x^2+2x+1}$ *(Damietta 17)*

3 **In each of the following , show whether $n_1 = n_2$ or not (give reason) :**

(1) 📖 $n_1(x) = \dfrac{x-1}{x}$, $n_2(x) = \dfrac{(x-1)(x^2+1)}{x(x^2+1)}$

(2) 📖 $n_1(x) = \dfrac{2x^3+6x}{(x-1)(x^2+3)}$, $n_2(x) = \dfrac{2x}{x-1}$

(3) $n_1(x) = \dfrac{x+5}{x^2-25}$, $n_2(x) = \dfrac{2}{2x-10}$ *(Ismailia 02)*

4 **In each of the following , prove that $n_1 = n_2$:**

(1) $n_1(x) = \dfrac{3x}{3x-6}$, $n_2(x) = \dfrac{2x}{2x-4}$ *(Souhag 06)*

(2) $n_1(x) = \dfrac{x}{x^2-1}$, $n_2(x) = \dfrac{5x}{5x^2-5}$

(3) $n_1(x) = \dfrac{2x}{2x+4}$, $n_2(x) = \dfrac{x^2+2x}{x^2+4x+4}$ *(El-Menia 17)*

(4) 🕮 $n_1(x) = \dfrac{x^3-1}{x^3+x^2+x}$, $n_2(x) = \dfrac{(x-1)(x^2+1)}{x^3+x}$

(5) $n_1(x) = \dfrac{x^2-x}{x^3-2x^2}$, $n_2(x) = \dfrac{x^2-3x+2}{x^3-4x^2+4x}$ *(Beni Suef 08)*

(6) 🕮 $n_1(x) = \dfrac{x^2}{x^3-x^2}$, $n_2(x) = \dfrac{x^3+x^2+x}{x^4-x}$ *(El-Beheira 17)*

(7) 🕮 $n_1(x) = \dfrac{x^3+x}{x^3+x^2+x+1}$, $n_2(x) = \dfrac{x}{x+1}$

Homework

Choose the correct answer :

1 n_1 , n_2 , n_3 and n_4 are four functions where $n_1(x) = x$, $n_2(x) = \dfrac{x^2}{x}$, $n_3(x) = \dfrac{x(x^2+4)}{(x^2+4)}$, $n_4(x) = \dfrac{x+5}{x^2}$, then the two equal functions are .... *(Damietta 03)*

(a) n_1 , n_2 (b) n_1 , n_3 (c) n_1 , n_4 (d) n_2 , n_3

---

2 If $p(x) = \dfrac{x^2-2x}{(x+2)(x-2)}$, $q(x) = \dfrac{x}{x+2}$, then p = q when .... *(Sharkia 03)*

(a) $p(x) = q(x)$ for each $x \in \mathbb{R} - \{-2\}$

(b) $p(x) = q(x)$ in the simplest form

(c) $p(x) = q(x)$ for each $x \in \mathbb{R} - \{2, -2\}$

(d) $p(x) = q(x)$ for each $x \in \mathbb{R}$

Complete:

1 If $x \neq 2$, then the simplest form of the fraction n where $n(x) = \frac{2-x}{x-2}$ is ....

2 The simplest form of the function n where $n(x) = \frac{4x^2-2x}{2x}$, $x \neq 0$ is ....

3 If $n_1(x) = \frac{x+1}{x-2}$, $n_2(x) = \frac{x^2+x}{x^2-2x}$, then the common domain in which $n_1 = n_2$ is .... *(Kafr El-Sheikh 11)*

4 If $n_1(x) = \frac{x}{x^2+x}$, $n_2(x) = \frac{1}{x+1}$, then $n_1 = n_2$ when $x \in$ .... *(New Valley 09)*

5 If $n_1(x) = \frac{1+a}{x-2}$, $n_2(x) = \frac{4}{x-2}$ and $n_1(x) = n_2(x)$, then a = ...

6 If the simplest form of the algebraic fraction $n(x) = \frac{x(x-2)}{x+a}$, $x \neq 2$ is $n(x) = x$, then a = ...

7 If the simplest form of the algebraic fraction $n(x) = \frac{x^2-4x+4}{x^2-a}$ is $n(x) = \frac{x-2}{x+2}$, then a = ...

8 **In each of the following , if n_1 and n_2 are two algebraic fractions , is $n_1 = n_2$? Why ?**

1 $n_1(x) = \frac{2x^2+4}{x^3+2x}$, $n_2(x) = \frac{4x^2+8}{2x^3+4x}$

2 $n_1(x) = \frac{x^2-2x}{x^2+x-6}$, $n_2(x) = \frac{x^2-3x}{x^2-9}$

Sheet (8)

Operations on the algebraic functions.

First Adding and subtracting the algebraic fractions :

1 Adding and subtracting two algebraic fractions having the same denominator :

If $x \in$ the common domain of the two algebraic fractions n_1 and n_2 where

$n_1(x) = \frac{f(x)}{k(x)}$ **and** $n_2(x) = \frac{p(x)}{k(x)}$ **, then :**

- $n_1(x) + n_2(x) = \frac{f(x)}{k(x)} + \frac{p(x)}{k(x)} = \frac{f(x) + p(x)}{k(x)}$
- $n_1(x) - n_2(x) = \frac{f(x)}{k(x)} - \frac{p(x)}{k(x)} = \frac{f(x) - p(x)}{k(x)}$

2 Adding and subtracting two algebraic fractions having different denominators :

If $x \in$ the common domain of the two algebraic fractions n_1 and n_2 where

$n_1(x) = \frac{f(x)}{r(x)}$ **and** $n_2(x) = \frac{p(x)}{k(x)}$ **, then :**

- $n_1(x) + n_2(x) = \frac{f(x)}{r(x)} + \frac{p(x)}{k(x)} = \frac{f(x) \times k(x) + p(x) \times r(x)}{r(x) \times k(x)}$
- $n_1(x) - n_2(x) = \frac{f(x)}{r(x)} - \frac{p(x)}{k(x)} = \frac{f(x) \times k(x) - p(x) \times r(x)}{r(x) \times k(x)}$

The steps of adding or subtracting two algebraic fractions :

1. Arrange the terms of each of the numerator and denominator of each fraction descendingly or ascendingly according to the powers of any variable in it.
2. Factorize the numerator and the denominator of each fraction if possible.
3. Find the common domain which will be the domain of the result.
4. Reduce each fraction separately to make the operations of addition or subtraction easier.
5. Unify the denominators.
6. Perform the operations of addition or subtraction of the terms of the numerators.
7. Put the final result in the simplest form if possible.

The properties of the operations of the addition and subtraction of the algebraic fractions :

- **The addition operation of the algebraic fractions has the following properties :**

1 Commutation. **2** Association.

3 Zero is the additive neutral (additive identity) of any algebraic fraction.

4 The additive inverse of any algebraic fraction is available.

i.e. the additive inverse of the algebraic fraction : $\frac{g(X)}{k(X)}$ is $-\frac{g(X)}{k(X)}$, $\frac{-g(X)}{k(X)}$ or $\frac{g(X)}{-k(X)}$

Choose the correct answer :

1 If $\frac{a}{b}$, $\frac{c}{d}$ are two algebraic fractions then $\frac{a}{b}+\frac{c}{d}=$ .... *(Beni Suef 2004)*

(a) $\frac{a+c}{b+d}$ (b) $\frac{ac}{bd}$ (c) $\frac{a+c}{bd}$ (d) $\frac{ad+bc}{bd}$

2 $\frac{2X}{3y}+\frac{5X}{4y}=$ .... in the simplest form *(Qena 2009)*

(a) $\frac{X}{y}$ (b) $\frac{7X}{12y^2}$ (c) $\frac{5X}{6y}$ (d) $\frac{23X}{12y}$

3 The domain of n : $n(X)=\frac{3X+4}{X^2+25}+\frac{X-2}{X^2+7}$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R}-\{5\}$

(c) $\mathbb{R}-\{-5,5\}$ (d) $\mathbb{R}-\{-5,5,-7\}$

4 $\frac{X}{X+1}+\frac{1}{X+1}=$ .... where $X\neq-1$

(a) $\frac{2X}{X+1}$ (b) $\frac{X}{X+1}$ (c) 1 (d) 2

5 If : $X\in\mathbb{R}-\{2\}$, then : $\frac{X}{X-2}+\frac{2}{2-X}=$ .... (In the simplest form) *(Beni Suef 2012)*

(a) 2 (b) 1 (c) -2 (d) -1

6 The additive inverse of the fraction $\frac{X+7}{X-5}$ is .... *(Fayoum 2012)*

(a) $\frac{7-X}{X+5}$ (b) $\frac{X+7}{5-X}$ (c) $\frac{-(X+7)}{5-X}$ (d) $\frac{X-7}{5-X}$

7 The domain of the additive inverse of the fraction $\frac{x+5}{x-7}$ is .... *(Souhag 2008)*

(a) $\mathbb{R}-\{5\}$ (b) $\mathbb{R}$ (c) $\mathbb{R}-\{7\}$ (d) $\mathbb{R}-\{5, 7\}$

8 If $n(x)=\frac{x}{x-3}-\frac{1}{x-3}$, then the set of zeroes of the function n is .... *(Helwan 2011)*

(a) $\{3\}$ (b) $\{1\}$ (c) $\{-1\}$ (d) $\{-3\}$

9 If the domain of n : $n(x)=\frac{3x}{(x-a)(x-2)}+\frac{2x-1}{(x-a)(x-3)}$ is $\mathbb{R}-\{3, 5, 2\}$

, then a $\in$ ....

(a) $\{5\}$ (b) $\{2, 3\}$ (c) $\mathbb{R}-\{5\}$ (d) $\mathbb{R}-\{2, 3, 5\}$

Essay problems:

1 **Find n (x) in its simplest form , and identify its domain where :**

$$n(x)=\frac{2x}{x+3}+\frac{6}{3+x}$$

2 **Find the function n (x) in its simplest form , showing its domain where :**

$$n(x)=\frac{x^2-x}{x^2-1}+\frac{1}{x+1}$$

3 **Find n (x) in its simplest form showing its domain where :**

$$n(x)=\frac{x^2+2x+4}{x^3-8}+\frac{x^2+x-2}{x^2-4}$$

4 **Find n (x) in the simplest form showing the domain of the function where :**

$$n(x)=\frac{x}{x-4}-\frac{x+4}{x^2-16}$$

5 **Find n (x) in the simplest form showing the domain of n where :**

$$n(x)=\frac{2x+6}{x^2+x-6}-\frac{x^2-6x}{x^2-8x+12}$$

Homework

Choose the correct answer :

1 $\frac{x}{x+1} + \frac{1}{x+1} = \dots\dots$ where $x \neq -1$

(a) $\frac{2x}{x+1}$ (b) $\frac{x}{x+1}$ (c) 1 (d) 2

2 The domain of n : $n(x) = \frac{3x+4}{x^2+25} + \frac{x-2}{x^2+7}$ is $\dots\dots$

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{5\}$

(c) $\mathbb{R} - \{-5, 5\}$ (d) $\mathbb{R} - \{-5, 5, -7\}$

3 If $n(x) = \frac{x}{x-3} - \frac{1}{x-3}$, then the set of zeroes of the function n is $\dots\dots$ *(Helwan 2011)*

(a) $\{3\}$ (b) $\{1\}$ (c) $\{-1\}$ (d) $\{-3\}$

4 $\frac{2x}{3y} + \frac{5x}{4y} = \dots\dots$ in the simplest form *(Qena 2009)*

(a) $\frac{x}{y}$ (b) $\frac{7x}{12y^2}$ (c) $\frac{5x}{6y}$ (d) $\frac{23x}{12y}$

5 **If : $x \in \mathbb{R} - \{2\}$, then : $\frac{x}{x-2} + \frac{2}{2-x} = \dots\dots$ (In the simplest form)** *(Beni Suef 2012)*

(a) 2 **(b) 1** **(c) -2** **(d) -1**

6 If $\frac{a}{b}$, $\frac{c}{d}$ are two algebraic fractions then $\frac{a}{b} + \frac{c}{d} = \dots\dots$ *(Beni Suef 2004)*

(a) $\frac{a+c}{b+d}$ (b) $\frac{ac}{bd}$ (c) $\frac{a+c}{bd}$ (d) $\frac{ad+bc}{bd}$

7 If the domain of n : $n(x) = \frac{3x}{(x-a)(x-2)} + \frac{2x-1}{(x-a)(x-3)}$ is $\mathbb{R} - \{3, 5, 2\}$

, then $a \in \dots\dots$

(a) $\{5\}$ (b) $\{2, 3\}$ (c) $\mathbb{R} - \{5\}$ (d) $\mathbb{R} - \{2, 3, 5\}$

8 The additive inverse of the fraction $\frac{x+7}{x-5}$ is $\dots\dots$ *(Fayoum 2012)*

(a) $\frac{7-x}{x+5}$ (b) $\frac{x+7}{5-x}$ (c) $\frac{-(x+7)}{5-x}$ (d) $\frac{x-7}{5-x}$

9 The domain of the additive inverse of the fraction $\frac{x+5}{x-7}$ is ………. *(Souhag 2008)*

(a) $\mathbb{R}-\{5\}$ (b) $\mathbb{R}$ (c) $\mathbb{R}-\{7\}$ (d) $\mathbb{R}-\{5, 7\}$

Essay problems:

1 **Find n in its simples form showing its domain where :** $n(x)=\frac{x^2-2x}{x^2-4}+\frac{2x+6}{x^2+5x+6}$

2 **Find n (x) in the simplest from showing the domain of n where :**

$$n(x)=\frac{x}{x(x+2)}+\frac{x-2}{x^2-4}$$

3 **Find n in its simplest form showing its domain , where :**

$$n(x)=\frac{x^2-2x}{x^2-4}+\frac{2x+6}{x^2+5x+6}$$

4 **Find in the simplest form :** $n(x)=\frac{x^2+2x+4}{x^3-8}+\frac{x^2+x-2}{x^2-4}$ and determine its domain.

5 **Find n (x) in the simplest form showing its domain where :**

$$n(x)=\frac{x+1}{x^2+3x+2}-\frac{x-2}{x^2-4}$$

6 **Find n (x) in the simplest form showing its domain where :**

$$n(x)=\frac{x^2}{x-1}+\frac{x}{1-x}$$

Sheet (9)

Operations on the algebraic functions (follow).

Second Multiplying and dividing the algebraic fractions :

1 Multiplying the algebraic fractions :

Remark

Notice the reduction of the numerator of the first number with the denominator of the second number and the numerator of the second number with the denominator of the first number.

- The following shows how to multiply two algebraic fractions :

Multiplying two algebraic fractions

If $x \in$ the common domain of the two algebraic fractions n_1 and n_2 where :

$n_1(x) = \frac{f(x)}{r(x)}$, $n_2(x) = \frac{p(x)}{k(x)}$

, then : $n_1(x) \times n_2(x) = \frac{f(x)}{r(x)} \times \frac{p(x)}{k(x)} = \frac{f(x) \times p(x)}{r(x) \times k(x)}$

The steps of multiplying the algebraic fractions :

1. Arrange the terms of each of the numerator and the denominator of each fraction alone descendingly or ascendingly according to the powers of any symbol in it.
2. Factorize the numerator and the denominator of each fraction alone if it is possible.
3. Find the common domain.
4. Remove the common factors between the numerator and the denominator of each fraction and between the numerator of a fraction and the denominator of another fraction.
5. Perform the operation of multiplication and put the result in the simplest form.

The properties of the operation of multiplying the algebraic fractions :

The operation of multiplying the algebraic fractions has the following properties :

1. Commutation.
2. Association.
3. One is the multiplicative neutral (the multiplicative identity).
4. Existing the multiplicative inverses.

The multiplicative inverse of the algebraic fraction :

If n is an algebraic fraction where $n(X) = \frac{p(X)}{k(X)} \neq 0$

, then n has a multiplicative inverse which is the algebraic fraction n^{-1} where $n^{-1}(X) = \frac{k(X)}{p(X)}$

and the domain of n^{-1} is $\mathbb{R}$ – the set of zeroes of each of the numerator and the denominator of any of the two fractions.

2 Dividing an algebraic fraction by another :

Dividing an algebraic fraction by another :

If n_1 and n_2 are two algebraic fractions where :

$n_1(X) = \frac{f(X)}{r(X)}$, $n_2(X) = \frac{p(X)}{k(X)}$, then : $n_1(X) \div n_2(X) = n_1(X) \times n_2^{-1}(X) = \frac{f(X)}{r(X)} \times \frac{k(X)}{p(X)}$

where the domain of $n_1 \div n_2$ = the common domain of each of n_1 , n_2 and n_2^{-1}

= $\mathbb{R}$ – the set of zeroes of the denominator of n_1 or the denominator of n_2

or the numerator of n_2

$= \mathbb{R} - \{z(r) \cup z(p) \cup z(k)\}$

Choose the correct answer :

1 The fraction $n(X) = \frac{X-2}{X}$ has a multiplicative inverse in the domain .... *(Cairo 2008)*

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{2\}$ (c) $\mathbb{R} - \{0\}$ (d) $\mathbb{R} - \{0, 2\}$

2 If $n(X) = \frac{X-1}{X-3}$, then the domain of n^{-1} is .... *(Beni Suef 2012)*

(a) $\mathbb{R} - \{3\}$ (b) $\mathbb{R} - \{1\}$ (c) $\mathbb{R} - \{1, 3\}$ (d) $\{1, 3\}$

3 If $n(X) = \frac{X^2 - 5X + 6}{5X}$, then the domain of n^{-1} is .... *(Ismailia 2003)*

(a) $\mathbb{R} - \{0\}$ (b) $\{0, 2, 3\}$

(c) $\mathbb{R} - \{0, 2, 3\}$ (d) $\mathbb{R} - \{5, -2, -3\}$

4 If $f(X) = \frac{X-2}{X+1}$, then $f^{-1}(2)$ is .... *(Menia 2009)*

(a) undefined (b) equal to 2 (c) zero (d) equal to – 1

5 If $n(X) = X + \frac{1}{X}$ where $X \neq 0$, then $n^{-1}(X) = \dots\dots\dots$ *(Port Said 2006)*

(a) $\frac{1}{X} + X$ (b) $\frac{1}{X+1}$ (c) $\frac{X}{X^2+1}$ (d) $-X - \frac{1}{X}$

6 The fraction $n(X) = \frac{X-2}{X}$ has a multiplicative inverse in the domain $\dots\dots\dots$ *(Cairo 2008)*

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{2\}$ (c) $\mathbb{R} - \{0\}$ (d) $\mathbb{R} - \{0, 2\}$

7 If $n(X) = \frac{X+2}{X-3}$, then the domain of n^{-1} is $\dots\dots\dots$ *(El-monofia 2014)*

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{3\}$ (c) $\mathbb{R} - \{-2\}$ (d) $\mathbb{R} - \{-2, 3\}$

Essay problems:

1 **Find n (X) in its simplest form, identify its domain where :**

$$n(X) = \frac{X^3+8}{X^2+5X+6} \times \frac{3X^2+9X}{X^2-2X+4}$$

2 **Find n (X) is the simplest form where :** $n(X) = \frac{X^2-3X+2}{X^2-1} \div \frac{X-5}{X^2-4X-5}$

Showing the domain of n (X)

3 **Find n (X) in the simplest form showing the domain of n where :**

$$n(X) = \frac{X^2-1}{X^2+3X+2} \div \frac{X^2-X}{X^2+2X}$$

4 If : $f(X) = \frac{X+1}{X^2-X-2} \times \frac{X^2+3X-10}{(3X+1)(X+5)}$,

then find : $f(X)$ in the simplest form and identify its domain

5 If : $n(X) = \frac{X^3-8}{X^3-7X^2+10X} \div \frac{X^2+2X+4}{3X^2-15X}$

Find : n (X) in the simplest from showing its domain.

6 If : $n(X) \frac{X^2-3X}{X^2-9} \div \frac{2X}{X+3}$ find n in its simplest form showing its domain.

7 **Find n in its simplest form , showing its domain where :**

$$n(X) = \frac{X^3 - 1}{X^2 - 2X + 1} \times \frac{2X - 2}{X^2 + X + 1}$$

8 If : $f(X) = \frac{X + 1}{X^2 - X - 2} \times \frac{X^2 + 3X - 10}{3X^2 + 16X + 5}$

, then find $f(X)$ in the simplest form and showing its domain.

9 **Find in the simplest form :** $n(X) = \frac{X^2 - 1}{X^2 + 3X + 2} \div \frac{X^2 - X}{X^2 + 2X}$ showing its domain.

10 **Find in the simplest form :** $f(X) = \frac{3X + 6}{X^2 - 4X - 5} \times \frac{X^2 - X - 20}{7X + 14}$ and show its domain.

11 **Find n (X) in the simplest form showing the domain of n :**

$$n(X) = \frac{X^2 - 3X}{X^2 - 9} \div \frac{2X}{X + 3}$$

Sheet (10)

Probability - Operations on events

- We can calculate the probability of an event (say A) from the relation :

$$P(A) = \frac{\text{The number of elements of the event A}}{\text{The number of elements of the sample spaces}} = \frac{n(A)}{n(S)}$$

For example:

In the experiment of rolling a fair die once and observing the number appears on the upper face , if S is the sample space of the experiment and A is the event of getting an even number , then :

$S = \{1 , 2 , 3 , 4 , 5 , 6\}$, $n(S) = 6$, $A = \{2 , 4 , 6\}$, $n(A) = 3$

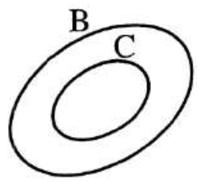
, then $P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$ (i.e. The probability of occurring the event $A = \frac{1}{2}$)

Remarks

- Zero ≤ the probability of any event ≤ 1
- Probability can be written as a fraction or percentage.

Remarks

From the previous example we notice that :



1 $C \subset B$ therefore $B \cap C = C$, then we deduce that :

The probability of occurring the two events B and C together

= the probability of occurring the event C

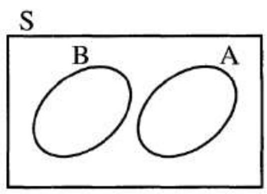
i.e. $P(B \cap C) = P(C) = \frac{n(C)}{n(S)}$

2 $A \cap C = \varnothing$ therefore it is said that the two events A and C are two mutually exclusive events

, then we can deduce that :

The probability of occurring the event A or C = $P(A \cup C) = P(A) = \frac{n(A)}{n(S)}$

Mutually exclusive events



- It is said that the two events A and B are mutually exclusive if

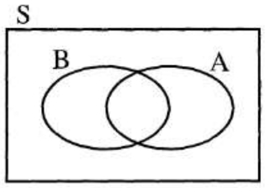
$A \cap B = \varnothing$, then $P(A \cap B) = 0$

i.e. The probability of their occurring together = the probability of the impossible event = 0

Rule :

- For any two events from the sample space S of a random experiment :

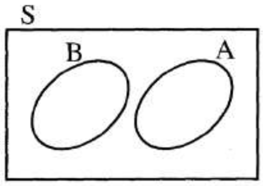
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



- If A and B are two mutually exclusive events , then :

$P(A \cap B) = \text{zero}$, then :

$$P(A \cup B) = P(A) + P(B)$$



Remarks

For any event A of the sample space S it will be :

1 $A \cap \grave{A} = \varnothing$

i.e. The two events A and $\grave{A}$ are two mutually exclusive events

i.e. Occurring one of them prevents the occurring of the other , then $P(A \cap \grave{A}) = \text{zero}$

2 $A \cup \grave{A} = S$

i.e. The union of any event and the complementary event of it = the set of sample space S ,

then $P(A \cup \grave{A}) = P(A) + P(\grave{A}) = P(S) = 1$

From that we deduce that :

$$P(A) = 1 - P(\grave{A}) \ , P(\grave{A}) = 1 - P(A)$$

Note that :

$$P(S) = \frac{n(S)}{n(S)} = 1$$

Remarks

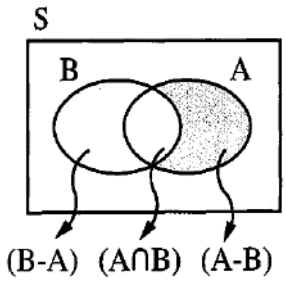
If A and B are two events of a sample space (S) of a random experiment ,

then $(A - B) \cup (A \cap B) = A$

i.e. $P(A - B) + P(A \cap B) = P(A)$

Also : $(B - A) \cup (A \cap B) = B$

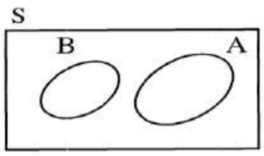
i.e. $P(B - A) + P(A \cap B) = P(B)$



Remarks

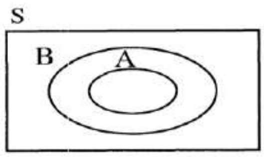
1 If A and B are two mutually exclusive of the sample space (S) , then :

- $A - B = A$ *i.e.* $P(A - B) = P(A)$
- $B - A = B$ *i.e.* $P(B - A) = P(B)$



2 If A and B are two events of the sample space (S) and $A \subset B$, then :

- $A - B = \varnothing$
- $P(A - B) = P(\varnothing) = \frac{n(\varnothing)}{n(S)} = \text{zero.}$



Choose the correct answer :

1 If A and B are two mutually exclusive events , then $P(A \cap B)$ equals .... *(Cairo 2014)*

(a) zero (b) $\varnothing$ (c) $P(A)$ (d) 1

2 If $A \subset B$, then $P(A \cup B)$ equals .... *(El-Dakahlia 2013)*

(a) zero (b) $P(A)$ (c) $P(B)$ (d) $P(A \cap B)$

3 If a regular coin is tossed once, then the probability of getting head or tail is .... *(Alexandria 2014 , El-Dakahlia 2013)*

(a) 100 % (b) 50 % (c) 25 % (d) zero %

4 If a regular die is rolled once, then the probability of getting an odd number and even number together equals .... *(El- Beheira 2014, Fayoum 2012)*

(a) zero (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) 1

5 A regular die is rolled once, if the event A is "appearing a prime number" and the event B is "appearing an odd number", then $P(A \cap B) =$ .... *(El-Sharkia 2011)*

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

6 If $P(A) = 4\,P(\grave{A})$, then $P(A) =$ .... *(Suez 2013)*

(a) 0.8 (b) 0.6 (c) 0.4 (d) 0.2

7 If A and B are two mutually exclusive in a random experiment and $P(\grave{A}) = 0.6$, $P(A \cup B) = 0.9$, then $P(B) =$ .... *(Kafr El-Sheikh 2013)*

(a) 0.5 (b) 0.4 (c) 0.6 (d) 0.3

8 If A and B are two events of the sample space of a random experiment , $P(A) = 0.6$ and $P(A \cap B) = 0.4$, then $P(A - B) =$ .... *(El-Wadi El-Gedied 2014)*

(a) 0.6 (b) 0.4 (c) 0.2 (d) 0.1

9 For any two events C and D of a random experiment There is : $(C - D) \cup (C \cap D) =$ .... *(El-Dakahlia 2014)*

(a) 1 (b) S (c) D (d) C

10 📖 If A and B are two mutually exclusive events , then $P(A \cap B)$ equals ……… *(Giza 2011)*

(a) $\varnothing$ (b) zero (c) 0.56 (d) 1

11 📖 If $A \subset B$, then $P(A \cup B)$ equals ……… *(Gharbia 2012)*

(a) zero (b) $P(A)$ (c) $P(B)$ (d) $P(A \cap B)$

12 📖 If a regular coin is tossed once, then the probability of getting head or tail is ……… *(Matrouh 2011)*

(a) zero % (b) 25% (c) 50% (d) 100%

13 📖 If a regular die is rolled once, then the probability of getting an odd number and even number together equals ……… *(Fayoum 2012)*

(a) zero (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) 1

14 A regular die is rolled once, if the event A is "appearing a prime number" and the event B is "appearing an odd number", then $P(A \cap B) =$ ……… *(El-Sharkia 2011)*

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

15 The probability of the impossible event equals ……………

(a) $\varnothing$ (b) zero (c) $\frac{1}{2}$ (d) 1

16 If the probability that a student in preparatory final exam is succeeded equals 85% , then the probability that he fail is ……………

(a) 0.015 (b) $\frac{3}{20}$ (c) $\frac{17}{20}$ (d) 0.85

17 If A and B are two events in a random experiment , and $A \subset B$, then $P(A \cup B) =$ ……………

(a) $P(A)$ (b) $P(B)$ (c) 0.5 (d) zero

18 If a coin is tossed once , then the probability that the head appears = ……………

(a) $\varnothing$ (b) 1 (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

Essay problems:

1 If A and B are two events in the sample space for a random experiment where $P(A) = 0.7$, $P(B) = 0.6$ and $P(A \cup B) = 0.9$, **then find :**

(1) $P(\grave{B})$ (2) $P(A \cap B)$

2 If A and B are two events of the sample space of a random experiment and there is :

$P(A) = \frac{1}{3}$, $P(B) = \frac{1}{2}$, $P(A \cap B) = \frac{1}{5}$ **Find :** $P(A \cup B)$

3 A ball is drawn randomly from 25 identical balls of the same volume and weight , 10 of them are red , 8 are white and the rest are green.

Find the probability that the drawn ball is : (1) white. (2) green or white.

4 If A and B are two events in the sample space of a random experiment and

$P(A) = 0.4$, $P(B) = 0.5$, $P(A \cup B) = 0.6$, **then find :** $P(A \cap B)$

5 A ball is drawn randomly from 25 identical balls of the same volume and weight ,

10 of them are red , 8 are white and the rest are green.

Find the probability that the drawn ball is :

(1) White. (2) Green or white. (3) Not green.

6 If A , B are two events from a sample space of a random experiment , and $P(A) = 0.8$, $P(\grave{B}) = 0.3$, $P(A \cup B) = 0.9$

Find : (1) $P(A \cap B)$ (2) $P(B - A)$

7 A bag contains 10 identical balls numbered from 1 to 10 , one ball is chosen randomly , if the event A is getting an odd number and the event B is getting a prime number.

Find : (1) $P(A)$ (2) $P(B)$ (3) $P(A - B)$

8 If A and B are two events from a sample space of the random experiment

, where $P(A) = \frac{1}{2}$, $P(\grave{B}) = \frac{1}{5}$, $P(A \cap B) = \frac{2}{5}$, **find :** $P(A \cup B)$

9 If A and B are two events in a sample space of a random experiment,
$P(A) = \frac{7}{10}$, $P(B) = \frac{3}{5}$, $P(A \cap B) = \frac{2}{5}$
Calculate : (1) $P(A \cup B)$ (2) Probability of non occurrence of event A

Homework

Essay problems:

1 If A and B are two events in a random experiment and if $P(A) = 0.2$, $P(B) = 0.6$, $P(A \cup B) = 0.5$
Find : (1) $P(A \cap B)$ (2) $P(\grave{A})$

2 If A and B are two events from a sample space of a random experiment and :
$P(A) = 0.2$, $P(B) = 0.6$, $P(A \cup B) = 0.5$, **then find :** $P(A \cap B)$

3 If A and B are two events from the sample space of a random experiment ,
and $P(A) = 0.5$, $P(A \cup B) = 0.8$ and $P(A \cap B) = 0.1$
Find : (1) $P(B)$ (2) $P(A - B)$

4 If A and B are two events in a random experiment, $P(A) = 0.6$, $P(B) = 0.7$
and $P(A \cap B) = 0.4$
Find : (1) $P(A \cup B)$ (2) $P(A - B)$

5 If A and B are two events from the sample space of a random experiment ,
if $P(A) = 0.3$, $P(A \cup B) = 0.7$, $P(B) = m$, **then find the value of m if :**
(1) A and B are two mutually exclusive events.
(2) $P(A \cap B) = 0.2$

6 If A and B are two events from the sample space of a random experiment ,
$P(A) = 0.3$, $P(B) = 0.6$, $P(A \cup B) = 0.7$ **Find :** $P(A \cap B)$

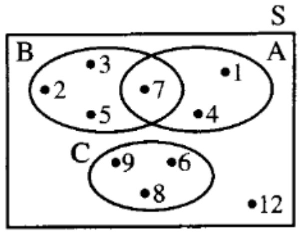
7 If A and B are two mutually exclusive events in a random experiment ,
$P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$, **find :** $P(A \cup B)$

8 Using the opposite "Venn diagram" :

Find : (1) n (S)

(2) P (A ⋂ B)

(3) P (C̀)



Sheet (11)

Accumulative Basic Skills

Choose the correct answer :

1 If the ratio between the perimeters of two squares is 1 : 2 , then the ratio between their areas = ... : ....

(a) 1 : 2 (b) 2 : 1 (c) 1 : 4 (d) 4 : 1

2 The equation : $3x - x^2 + 1 = 0$ is of .... degree.

3 If x is a negative number, then the greatest number of the following is ....

(a) $5x$ (b) $\frac{5}{x}$ (c) $5 + x$ (d) $5 - x$

4 If the sum of ages of a father and his sun now is 47 years , then the sum of their ages after 10 years = ... years.

(a) 27 (b) 37 (c) 57 (d) 67

5 $(x + 1)^2 =$ ....

(a) $x^2 + 1$ (b) $x^2 - 1$ (c) $x^2 - x + 1$ (d) $x^2 + 2x + 1$

6 If x is a negative real number , then the greatest number of the following numbers is ....

(a) $3 + x$ (b) $3x$ (c) $3 - x$ (d) $\frac{3}{x}$

7 If $x = 2$ and $y = 3$, then $(y - 2x)^{10} =$ ....

(a) 10 (b) -1 (c) -10 (d) 1

8 If $2^7 \times 3^7 = 6^k$, then k = ...

(a) 14 (b) 7 (c) 6 (d) 5

9 If $x^2 - y^2 = 2(x + y)$ where $(x + y) \neq$ zero , then $(x - y) =$ ....

(a) 2 (b) 4 (c) 6 (d) 8

10 If $2^5 \times 3^5 = m \times 6^4$, then m = ...

(a) 1 (b) 2 (c) 3 (d) 6

11 If $(7^{a-2} , 3) = (1 , b + 5)$, then a + b = ...

(a) – 1 (b) zero (c) 1 (d) 2

12 If $a < \sqrt{3} < b$, then (a , b) is ....

(a) (0 , 1) (b) (2.5 , 3.5) (c) (1 , 2) (d) (2 , 3)

13 If $2^8 \times 3^8 = x \times 6^8$, then x = ...

(a) 2 (b) 3 (c) 6 (d) 1

14 If the age of a man now is x year , then his age after 5 years from now is .... years.

(a) $x - 5$ (b) $5 - x$ (c) $5x$ (d) $x + 5$

15 If $(5 , x - 4) = (y , 3)$, then $x + y$ = ...

(a) 25 (b) 12 (c) 8 (d) 6

16 If $x - y = 2$, $x^2 - y^2 = 10$, then $x + y$ = ...

(a) – 5 (b) 2 (c) – 2 (d) 5

17 The degree of the equation : $3x + 4y + xy = 5$ is ....

(a) zero. (b) first. (c) second. (d) third.

Best wishes

GEOMETRY FOR

PREPARATORY THREE

SECOND TERM

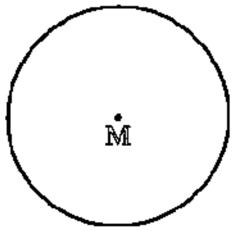
Sheet (1)

Basic definitions and concepts on the circle

The circle

It is the set of points of the plane which are at a constant distance from a fixed point in the same plane.

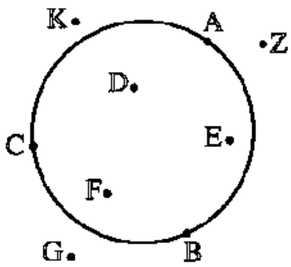
- The fixed point is called "**the centre of the circle**".
- The constant distance is called "**the radius length of the circle**".
- The circle is usually denoted by its centre , so we say the circle M to mean the circle whose centre is the point M



Partition of the plane by the circle

- **The drawn circle divides the plane into three sets of points as shown in the opposite figure :**

1. **The set of points on the circle as :** the points A , B , C , ...
2. **The set of points inside the circle as :** the points D , E , F , ...
3. **The set of points outside the circle as :** the points Z , K , G , ...



Notice that :

- The surface of the circle is the set of points on the circle ∪ the set of points inside it.
- There is a difference between the circle and the surface of the circle.

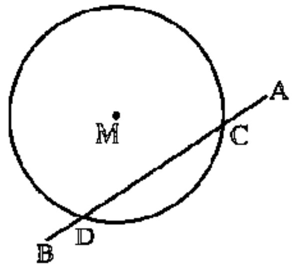
For example : **In the opposite figure :**

* $\overline{AB} \cap$ the circle $= \{C , D\}$

but $\overline{AB} \cap$ the surface of the circle $= \overline{CD}$

* $M \notin$ the circle

but $M \in$ the surface of the circle.



The radius of the circle

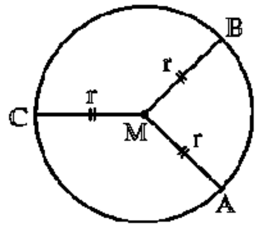
It is a line segment with one endpoint at the centre of the circle and the other endpoint on the circle.

In the opposite figure :

If the points A , B and C belong to the circle M ,

then $\overline{MA}$, $\overline{MB}$ and $\overline{MC}$ are called radii of the circle M

and MA = MB = MC = r (where r is the radius length of the circle).



Notice that :

1. Any circle has an infinite number of radii and all of them are equal in length.
2. If two radii of two circles are equal in length , then the two circles are congruent and vice versa.

The chord of the circle

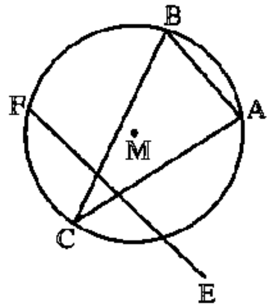
It is a line segment whose endpoints are any two points on the circle.

In the opposite figure :

If A , B and C belong to the circle

, then each of $\overline{AB}$, $\overline{AC}$ and $\overline{BC}$

is a chord of the circle M



Notice that :

$\overline{EF}$ is not a chord of the circle M because $E \notin$ the circle M

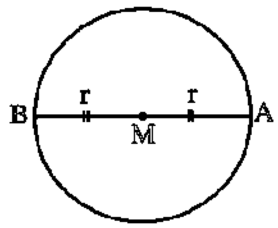
The diameter of the circle

It is a chord passing through the centre of the circle.

In the opposite figure :

If M is a circle , $\overline{AB}$ is a chord of it

, $M \in \overline{AB}$, then $\overline{AB}$ is a diameter of the circle M



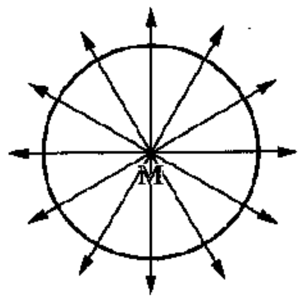
The circumference of the circle and its area

- The circumference of the circle $= 2\pi r$ length unit.
- The area of the circle $= \pi r^2$ square unit.

(where r is the radius length and π is the approximating ratio).

Symmetry in the circle

- Any straight line passing through the centre of the circle is an axis of symmetry of it.
- Since the number of these straight lines are infinite , then the circle has an infinite number of axes of symmetry.



Important Corollaries

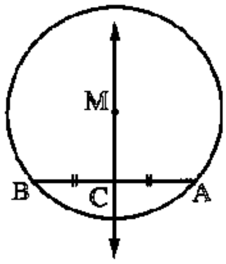
Corollary 1

The straight line passing through the centre of the circle and the midpoint of any chord of it (not passing through the centre) is perpendicular to this chord.

In the opposite figure :

If $\overline{AB}$ is a chord of the circle M

and C is the midpoint of $\overline{AB}$, then $\overleftrightarrow{MC} \perp \overline{AB}$

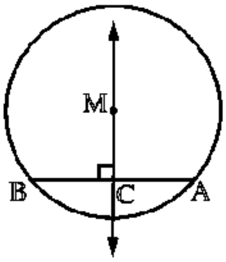


Corollary 2

The straight line passing through the centre of the circle and perpendicular to any chord of it bisects this chord.

In the opposite figure :

If $\overline{AB}$ is a chord of the circle M and $\overleftrightarrow{MC} \perp \overline{AB}$, where $C \in \overline{AB}$, then C is the midpoint of $\overline{AB}$



Corollary 3

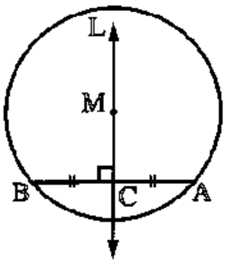
The perpendicular bisector to any chord of a circle passes through the centre of the circle.

In the opposite figure :

If $\overline{AB}$ is a chord of the circle M , C is the midpoint of $\overline{AB}$

and the straight line $L \perp \overline{AB}$ from the point C ,

then $M \in$ the straight line L



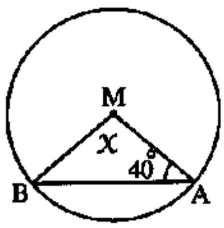
From the previous, we deduce that :

The axis of symmetry of any chord of a circle passes through its centre , so this axis is also an axis of symmetry of the circle.

Complete:

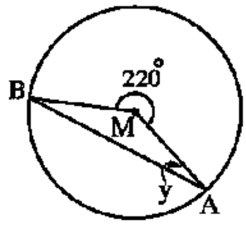
1 📖 **In each of the following figures , find the value of the used symbol in measuring where M is the centre of the circle :**

(1)



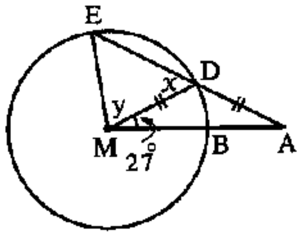
x = ...°

(2)



y = ...°

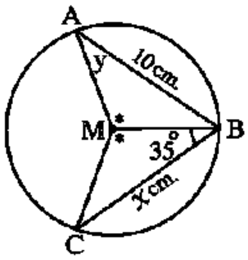
(3)



x = ...°

y = ...°

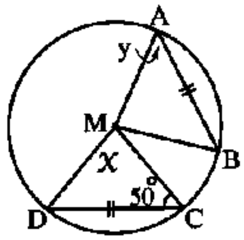
(4)



x = ... cm.

y = ...°

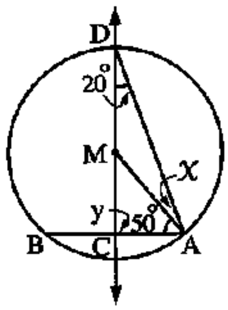
(5)



x = ...°

y = ...°

(6)

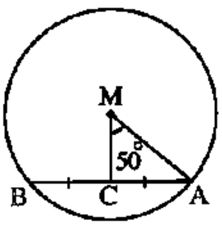


x = ...°

y = ...°

2 📖 **In each of the following figures , M is a circle , complete :**

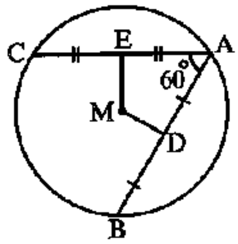
(1)



m (∠ MAC) = ...°

(Luxor 14 , Assiut 11)

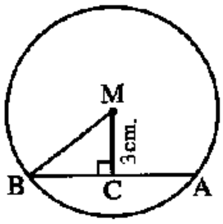
(2)



m (∠ DME) = ...°

(Luxor 14 , Giza 15)

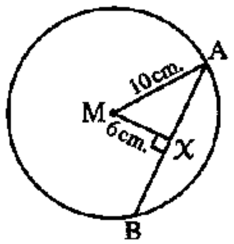
(3)



If AB = 8 cm.

, then MB = ... cm.

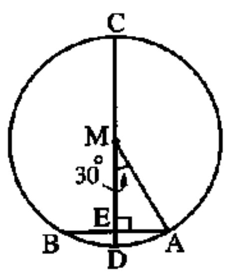
(4)



AB = ... cm.

(Red Sea 12)

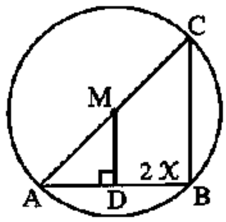
(5)



If AB = 10 cm.

, then CD = ... cm.

(6)



x = ...°

Essay problems:

1 **In the opposite figure :**

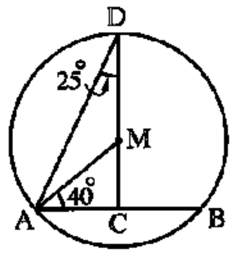
$\overline{AB}$ is a chord of the circle M ,

m ($\angle$ D) = 25°

and m ($\angle$ MAC) = 40°

Prove that :

C is the midpoint of $\overline{AB}$



(Kafr El-Sheikh 09)

2 📖 **In the opposite figure :**

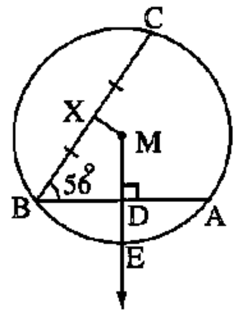
$\overline{AB}$ and $\overline{BC}$ are two chords in circle M ,

which has radius length of 5 cm. ,

$\overrightarrow{MD} \perp \overline{AB}$ intersects $\overline{AB}$ at D and inersects the circle M at E ,

X is the midpoint of $\overline{BC}$, AB = 8 cm. , m ($\angle$ ABC) = 56°

Find : (1) m ($\angle$ DMX) (2) The length of $\overline{DE}$



(El-Gharbia 17 , Souhag 15) « 124° , 2 cm. »

3 **In the opposite figure :**

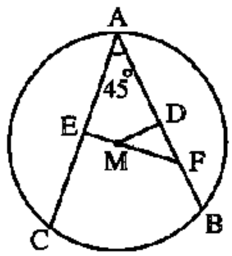
$\overline{AB}$ and $\overline{AC}$ are two chords of the circle M ,

m ($\angle$ BAC) = 45° ,

D and E are the midpoints

of $\overline{AB}$ and $\overline{AC}$ respectively.

Prove that : Δ DFM is an isosceles triangle.



(New Valley 05)

4 📖 **In the opposite figure :**

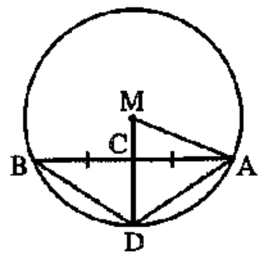
M is a circle of radius length 13 cm. ,

$\overline{AB}$ is a chord of length 24 cm. ,

C is the midpoint of $\overline{AB}$

and $\overrightarrow{MC} \cap$ circle M = {D}

Find : The area of the triangle ADB



(El-Dakahlia 13) « 96 cm.^{2} »

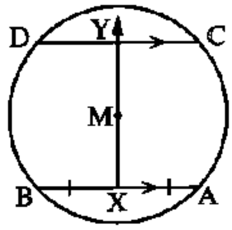
5 📖 **In the opposite figure :**

M is a circle , $\overline{AB}$ // $\overline{CD}$,

X is the midpoint of $\overline{AB}$

and $\overrightarrow{XM}$ is drawn to cut $\overline{CD}$ at Y

Prove that : Y is the midpoint of $\overline{CD}$



(Aswan 15 , Alexandria 13)

---

6 In a cartesian coordinates plane , if $\overline{AB}$ is a diameter of the circle M where A (3 , 4) and B (3 , – 3) , find the coordinates of M , then calculate the circumference of the circle.

« $(3 , \frac{1}{2})$, 22 length units »

---

7 In a cartesian coordinates plane , $\overline{AB}$ is a chord of the circle M , D is the midpoint of $\overline{AB}$

If A (4 , 1) and B (– 4 , 5) **Find :** The equation of $\overleftrightarrow{MD}$ « $y = 2x + 3$ »

---

8 In a cartesian coordinates plane , if M (– 1 , 2) , A (2 , 6) and B (2 , – 2)

Prove that M is the centre of a circle passing through the two points A and B , then calculate the perpendicular distance between the chord $\overline{AB}$ and the centre of the circle.

« 3 length units »

Homework

Essay problems:

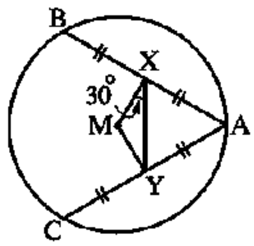
1 **In the opposite figure :**

AC = AB , X is the midpoint of $\overline{AB}$,

Y is the midpoint of $\overline{AC}$,

m (∠ MXY) = 30°

Prove that : The triangle AXY is equilateral.



(Assiut 14)

---

2 📖 **In the opposite figure :**

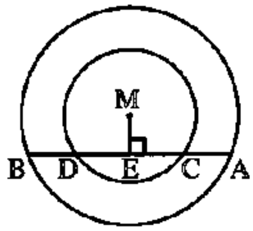
Two concentric circles with centre M ,

$\overline{AB}$ is a chord of the greater circle

and intersects the smaller circle at C , D

and $\overline{ME} \perp \overline{AB}$

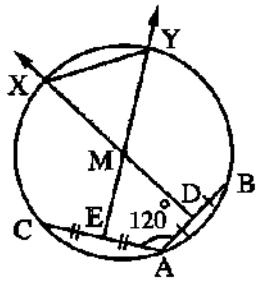
Prove that : AC = BD



(Qena 17 , Red Sea 12)

3 **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two chords in circle M that includes an angle of measure 120° , D and E are the two midpoints of $\overline{AB}$ and $\overline{AC}$ respectively , $\overrightarrow{DM}$ and $\overrightarrow{EM}$ are drawn to intersect the circle at X and Y respectively.

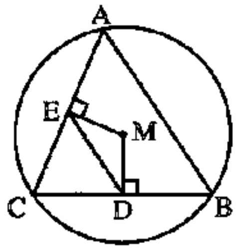


Prove that : The triangle XYM is an equilateral triangle.

(Aswan 16 , Beni Suef 15)

4 **In the opposite figure :**

ABC is a triangle drawn inside a circle with centre M (inscribed triangle) , $\overline{MD} \perp \overline{BC}$ and $\overline{ME} \perp \overline{AC}$



Prove that :

(1) $\overline{ED} \parallel \overline{AB}$

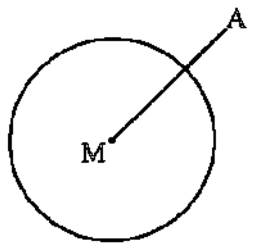
(Kafr El-Sheikh 16 , El-Beheira 13)

(2) The perimeter of Δ CDE = $\frac{1}{2}$ the perimeter of Δ ABC

Sheet (2)

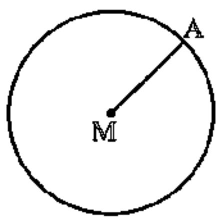
Position of a point and a straight line with respect to a circle

1 A is outside the circle M



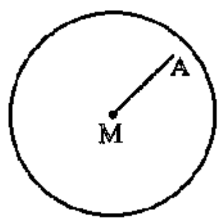
If MA > r

2 A is on the circle M

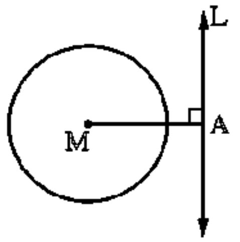
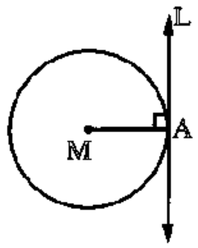
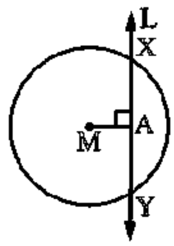


If MA = r

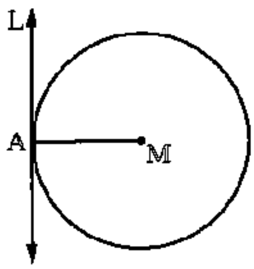
3 A is inside the circle M



If MA < r

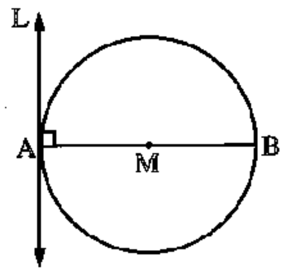
If	Then	The figure	Note that
1 **MA > r**	The straight line L lies outside the circle M		• L ∩ the circle M = ∅ • L ∩ the surface of the circle M = ∅
2 **MA = r**	The straight line L is a tangent to the circle M at A A is called "the point of tangency"		• L ∩ the circle M = {A} • L ∩ the surface of the circle M = {A}
3 **MA < r**	The straight line L is a secant to the circle M		• L ∩ the circle M = {X , Y} • L ∩ the surface of the circle M = $\overline{XY}$ • $\overline{XY}$ is called the chord of intersection

1 The tangent to a circle is perpendicular to the radius drawn from the point of tangency.



i.e. if the straight line L is a tangent to the circle M at the point A , then $\overline{MA} \perp L$

2 The straight line which is perpendicular to the diameter of a circle at one of its endpoints is a tangent to the circle.



i.e. if $\overline{AB}$ is a diameter of the circle M and the straight line L $\perp$ $\overline{AB}$ at the point A , then L is a tangent to the circle M at the point A

The two tangents which are drawn from the two endpoints of a diameter of a circle are parallel.

Choose the correct answer :

1 If M is a circle , its diameter = 6 cm. and A is a point on the circle , then ....

(a) MA > 6 cm. (b) MA = 6 cm.

(c) MA = 3 cm. (d) MA < 3 cm.

2 If the straight line L is a tangent to the circle whose length of diameter is 8 cm. , then the straight line is at a distance .... cm. from its centre. *(Aswan 2011)*

(a) 5 (b) 4 (c) 3 (d) 2

3 A circle M is of radius length 5 cm. , A is a point outside the circle , then MA equals .... cm. *(Gharbia 2003)*

(a) 3 (b) 5 (c) 8 (d) 4

4 M is a circle whose diameter length = 6 cm. If the straight line L is at a distance of 4 cm. from its centre , then the straight line L is .... *(Monofia 2008)*

(a) a secant to the circle. (b) a tangent to the circle.

(c) outside the circle. (d) passing through the centre of the circle.

5 M is a circle of radius length 5 cm. A is a point on the circle where MA = $(2X + 1)$ cm. , then X = ... *(El-Ismailia 2011)*

(a) 11 (b) – 2 (c) 1 (d) 2

6 🕮 $\overleftrightarrow{AB}$ is a diameter in a circle M , $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$ are two tangents to the circle , then $\overleftrightarrow{AC}$ .... $\overleftrightarrow{BD}$

(a) intersects (b) is perpendicular to

(c) is parallel to (d) is coincident to

7 🕮 A circle is of a circumference 6π cm. , and the straight line L is distant from its centre by 3 cm. , then the straight line L is .... *(New Valley 2012)*

(a) a tangent to the circle. (b) a secant.

(c) outside the circle. (d) a diameter of the circle.

8 If the area of the circle M is 16π cm^2. A is a point in its plane where MA = 8 cm. , then A lies .... the circle M *(Sharkia 2009)*

(a) inside (b) outside (c) on (d) at the centre of

9 M is a circle with diameter of length 8 cm. If the straight line L is outside the circle, then the distance between the centre of the circle and the straight line L $\in$ ....

(a) $[0, 4]$ (b) $]0, 4[$ (c) $[0, 4[$ (d) $]4, \infty[$ *(Dakahlia 2006)*

10 If the point A is in the plane of the circle M whose radius length = r and if $0 < MA < r$, then A lies .... *(Souhag 2006)*

(a) outside the circle. (b) inside the circle.

(c) on the circle. (d) at the centre of the circle.

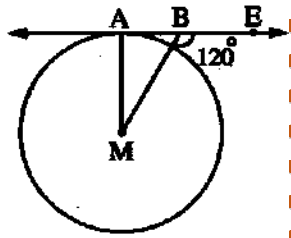
11 A circle with diameter length $2X$ cm., the straight line L is at a distance $(X + 1)$ cm. from its centre, then the straight line L is .... *(Fayoum 2012)*

(a) a secant to the circle. (b) outside the circle.

(c) a tangent to the circle. (d) passing through its centre.

12 If $\overleftrightarrow{AB}$ is a tangent to the circle M at A and m ($\angle$ MBE) = 120°, then m ($\angle$ AMB) = ... *(Souhag 2008)*

(a) 60° (b) 30° (c) 80° (d) 90°

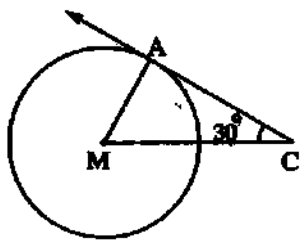


13 $\overleftrightarrow{CA}$ touches the circle M at A, m ($\angle$ ACM) = 30°

If the radius length of the circle M = 4 cm., then

MC = ... cm. *(Fayoum 2009)*

(a) 8 (b) 4 (c) $4\sqrt{3}$ (d) $8\sqrt{3}$



Essay problems:

1 **In the opposite figure :**

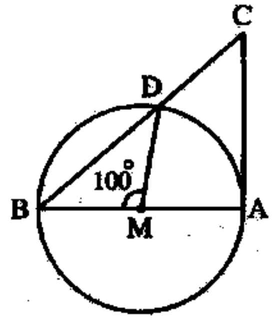
$\overline{AB}$ is a diameter in the circle M,

$\overrightarrow{AC}$ is a tangent to the circle at A,

m ($\angle$ DMB) = 100°

Find by proof :

1 m ($\angle$ ACB) 2 m ($\angle$ CDM) *(El-Menia 2011)* «50°, 140°»

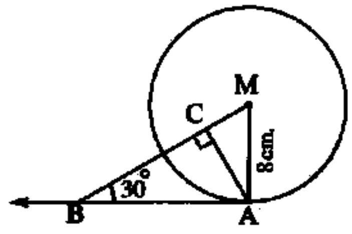


2 In the opposite figure :

$\overrightarrow{AB}$ is a tangent to the circle M at A ,

MA = 8 cm. , m ($\angle$ ABM) = 30° and $\overline{AC} \perp \overline{MB}$

Find : The length of each of $\overline{AB}$ and $\overline{AC}$

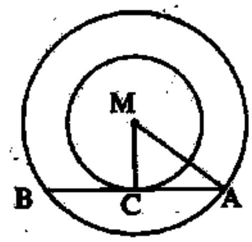


(New Valley 2012) « $8\sqrt{3}$ cm. , $4\sqrt{3}$ cm. »

3 In the opposite figure :

$\overline{AB}$ is a chord of the great circle and touches the small circle at C , AB = 8 cm. and the radius length of the great circle = 5 cm.

Find : The radius length of the small circle.



(Souhag 2009) «3 cm.»

Homework

Choose the correct answer :

1 M is a circle whose diameter length = 6 cm. If the straight line L is at a distance of 4 cm. from its centre , then the straight line L is .... *(Monofia 2008)*

(a) a secant to the circle. (b) a tangent to the circle.

(c) outside the circle. (d) passing through the centre of the circle.

2 If the area of the circle M is 16π cm^2. A is a point in its plane where MA = 8 cm. , then A lies .... the circle M *(Sharkia 2009)*

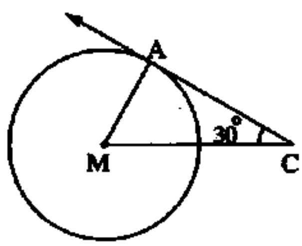
(a) inside (b) outside (c) on (d) at the centre of

3 $\overleftrightarrow{CA}$ touches the circle M at A , m ($\angle$ ACM) = 30°

If the radius length of the circle M = 4 cm. , then

MC = ... cm. *(Fayoum 2009)*

(a) 8 (b) 4 (c) $4\sqrt{3}$ (d) $8\sqrt{3}$



4 A circle M is of radius length 5 cm. , A is a point outside the circle , then MA equals .... cm. *(Gharbia 2003)*

(a) 3 (b) 5 (c) 8 (d) 4

5 🕮 A circle is of a circumference 6 π cm. , and the straight line L is distant from its centre by 3 cm. , then the straight line L is .... *(New Valley 2012)*

(a) a tangent to the circle. (b) a secant.

(c) outside the circle. (d) a diameter of the circle.

6 A circle with diameter length $2X$ cm. , the straight line L is at a distance $(X + 1)$ cm. from its centre , then the straight line L is .... *(Fayoum 2012)*

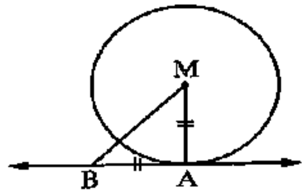
(a) a secant to the circle. (b) outside the circle.

(c) a tangent to the circle. (d) passing through its centre.

7 If $\overleftrightarrow{AB}$ is a tangent to the circle M at A ,

AB = AM , then m (∠ M) = ...

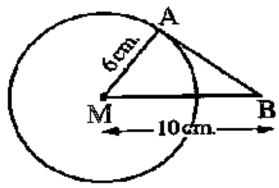
(a) 30° (b) 45° (c) 60° (d) 90°



8 If $\overline{AB}$ touches the circle M at A ,

AM = 6 cm. , MB = 10 cm. , then AB = ... cm.

(a) 6 (b) 8 (c) 10 (d) 12



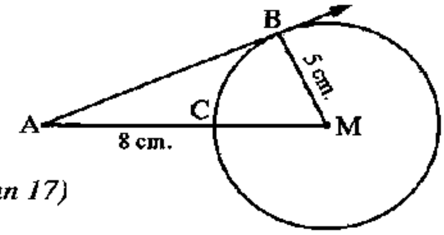
9 **In the opposite figure :**

$\overrightarrow{AB}$ is a tangent to circle M

If MB = 5 cm. , AC = 8 cm.

, then AB = ... cm. *(Kafr El-Sheikh 17 , Aswan 17)*

(a) 5 (b) 10 (c) 12 (d) 13

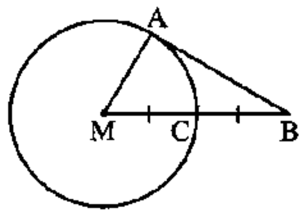


10 If $\overline{AB}$ touches the circle M at A ,

$\overline{MB} \cap$ the circle M = {C} where

MC = BC , then m (∠ B) = ...

(a) 30° (b) 45° (c) 60° (d) 90°



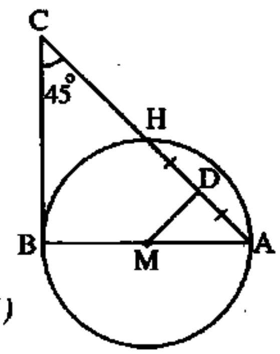
Essay problems:

1 **In the opposite figure :**

$\overrightarrow{BC}$ is a tangent at B , m (∠ C) = 45° ,

D is the midpoint of $\overline{AH}$

Prove that : DA = DM



(Aswan 2011)

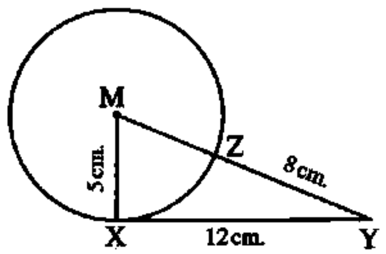
2 **In the opposite figure :**

M is a circle with radius length 5 cm. ,

XY = 12 cm. , $\overline{MY}$ ∩ circle M = {Z}

and ZY = 8 cm.

Prove that : $\overleftrightarrow{XY}$ is a tangent to the circle M at X



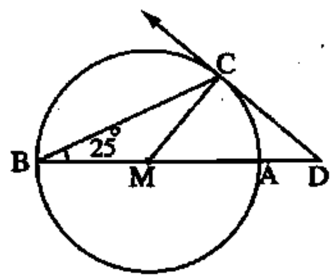
3 **In the opposite figure :**

$\overline{AB}$ is a diameter of the circle M ,

D ∈ $\overrightarrow{BA}$ If $\overrightarrow{DC}$ is a tangent to the circle at C

and m (∠ B) = 25°

Find : m (∠ D)



(Beni Suef 2003) «40°»

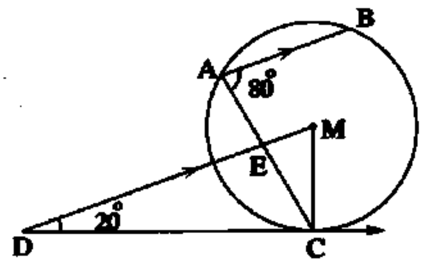
4 **In the opposite figure :**

$\overleftrightarrow{DC}$ touches the circle M at C , $\overline{AB}$ // $\overline{MD}$,

m (∠ BAC) = 80° , m (∠ MDC) = 20°

and $\overline{AC}$ ∩ $\overline{MD}$ = {E}

Find : m (∠ ECM)

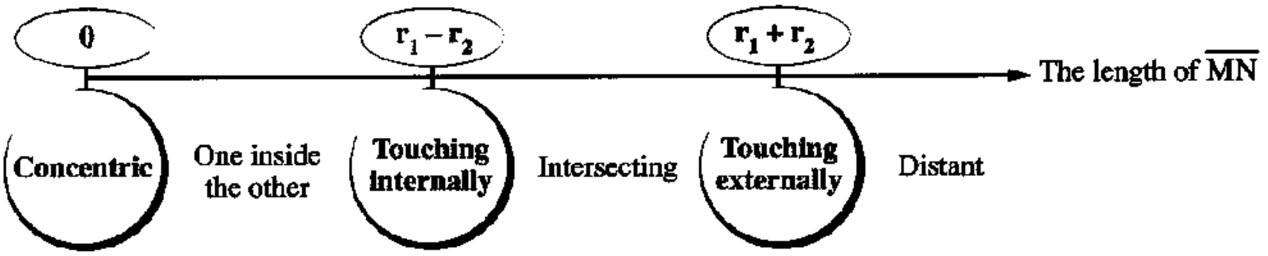


(Beni suef 2005) «30°»

Sheet (3)

Position of a circle with respect to another circle

Summary



From the previous summary , we notice that :

1. If M and N are two distant circles , then : $MN \in \,]r_1 + r_2 , \infty[$
2. If M and N are two intersecting circles , then : $MN \in \,]r_1 - r_2 , r_1 + r_2[$
3. If M and N (one of them is inside the other) , then : $MN \in \,]0 , r_1 - r_2[$

Complete:

1 **Let M and N be two circles , their radii lengths are 4 cm. and 9 cm. respectively.**
Complete the following :

1. If the two circles M and N are touching externally , then : MN ....
2. If the two circles M and N are touching internally , then : MN ....
3. If the two circles M and N are intersecting , then : MN ....
4. If the two circles M and N are concentric , then : MN ....
5. If the two circles M and N are distant , then : MN ....
6. If the two circles M and N are one of them is inside the other , then : MN ....

Choose the correct answer :

1 **M and N are two circles touching internally. If their radii lengths are 5 cm. and 2 cm., then MN = ... cm.** *(Damietta 2008)*

(a) zero (b) 3 (c) 7 (d) 10

2 M and N are two circles , their radii lengths are 4 cm. and 3 cm. If MN = 9 cm. , then the two circles are .... *(Port Said 2009)*

(a) distant. (b) intersecting.
(c) touching. (d) one is inside the other.

3 M and N are two intersecting circles their radii lengths are 3 cm. and 5 cm. respectively , then MN ∈ .... *(Alexandria , Suez , Assuit 2011)*

(a)]0 , 2[(b)]2 , 8[(c)]8 , ∞[(d)]2 , ∞[

4 If the radius length of the circle M = the radius length of the circle N = MN , then the two circles are .... *(Alex. 2005)*

(a) one is inside the other. (b) touching externally.
(c) distant. (d) intersecting.

5 M , N and L are three circles touching externally two-by-two, their radii length are 5 cm., 6 cm. and 4 cm., then the perimeter of the triangle MNL = ... *(El-Monafia 2011)*

(a) 15 cm. (b) 30 cm. (c) 4 cm. (d) 60 cm.

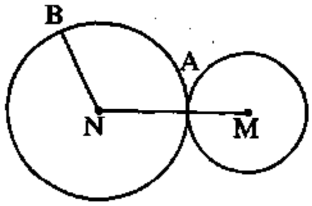
6 M is a circle of radius length 4 cm., touches the circle N externally. If MN = 7 cm., then the circumference of the circle N = ... cm. *(Beheira 2003)*

(a) 4 π (b) 6 π (c) 7 π (d) 8 π

Essay problems:

1 **In the opposite figure :**

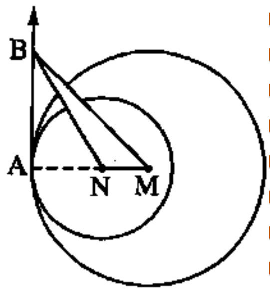
M and N are two circles touching at A ,
the distance between their centres MN = 12 cm.
If NB = 7 cm. ,
Find : The length of $\overline{MA}$



(Kafr El-Sheikh 2006) «5 cm.»

2 **In the opposite figure :**

M and N are two circles with radii lengths of 10 cm. and 6 cm. respectively and they are touching internally at A ,
$\overleftrightarrow{AB}$ is a common tangent for both.
If the area of Δ BMN = 24 cm^2 ,
Find : The length of $\overline{AB}$



(Port Said 2014) « 12 cm. »

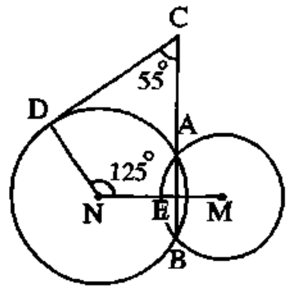
3 **In the opposite figure :**

M and N are two intersecting circles at A and B ,

C $\in \overrightarrow{BA}$, D $\in$ the circle N ,

m ($\angle$ MND) = 125° and m ($\angle$ BCD) = 55°

Prove that : $\overleftrightarrow{CD}$ is a tangent to circle N at D



(Souhag 2014 , El-Menia 2011)

4 **In the opposite figure :**

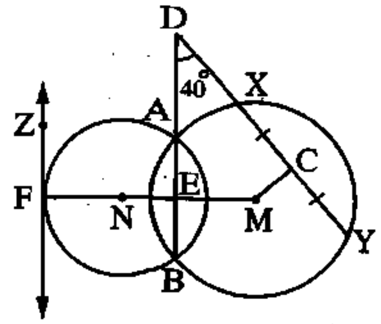
M and N are two intersecting circles at A and B ,

C is the midpoint of $\overline{XY}$, m ($\angle$ D) = 40° ,

$\overleftrightarrow{FZ}$ is a tangent to the circle N at F where $\overleftrightarrow{MN} \cap \overleftrightarrow{FZ} = \{F\}$

1 **Find :** m ($\angle$ CME)

2 **Prove that :** $\overleftrightarrow{FZ} \parallel \overline{AB}$



(El-Fayoum 2011) « 140° »

Homework

Choose the correct answer :

1 **M and N are two intersecting circles at A and B , then the axis of symmetry of $\overline{AB}$ is ....** *(Monofia 2004)*

(a) $\overline{MN}$ (b) $\overrightarrow{NM}$ (c) $\overleftrightarrow{MN}$ (d) $\overrightarrow{MN}$

2 M and N are two touching circles where MN = 6 cm. , the radius length of the greater circle is 10 cm. , then the radius length of the smaller circle = ... cm. *(Sharkia 2005)*

(a) 16   (b) 12   (c) 8   (d) 4

3 **M and N are two circles of radii lengths 3 cm. and 8 cm. respectively , MN = 5 cm. , then the two circles are ....** *(Cairo 2011)*

(a) touching externally.   (b) touching internally.

(c) distant.   (d) intersecting.

4 **If the radius length of the circle M = 3 cm. and the radius length of the circle N = 5 cm. , MN = 6 cm. , then the two circles M and N are ....** *(Gharbia 2008)*

(a) distant.   (b) one is inside the other.

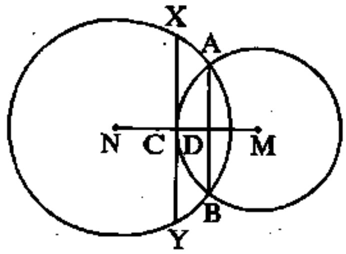
(c) intersecting.   (d) touching externally.

5 A circle M of radius length 4 cm. touches a circle N internally , MN = 7 cm. , then the circumference of the circle M : the circumference of the circle N = ...

(a) 4 : 7 (b) 3 : 4 (c) 4 : 3 (d) 4 : 11 *(Dakahlia 2009)*

Essay problems:

1 **In the opposite figure :**



M and N are two intersecting circles , $\overline{AB}$ is the common chord of the two circles M and N

$\overleftrightarrow{XY}$ touches the circle M at C

Prove that : $\overline{AB}$ // $\overleftrightarrow{XY}$ *(Souhag 2008)*

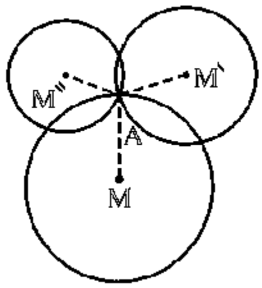
2 M and N are two intersecting circles at A and B , MA = 12 cm. , NA = 9 cm. and MN = 15 cm.

Find : The length of $\overline{AB}$ *(Port Said 2011)* «14.4 cm.»

Sheet (4)
Identifying the circle

First : Drawing a circle passing through a given point :

If A is a given point in the plane and the required is drawing a circle passing through the point A

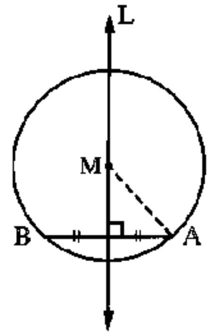


- Assume any other point in the plane as M , then take it as a centre using the compasses , draw a circle with the centre M and radius length = MA , then it will pass through the point A
- Similarly , we can draw another circle whose centre is M̀ and its radius length is M̀A , then it passes through the point A or we draw a circle whose centre is M̀̀ and its radius length = M̀̀A , then it will pass through the point A and so on ....

We can draw an infinite number of circles passing through a given point.

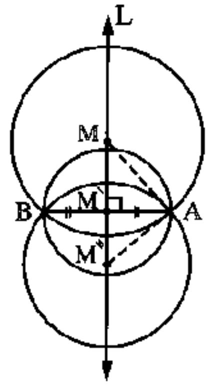
Second : Drawing a circle passing through two given points :

If A and B are two given points in the plane and the required is drawing a circle passing through the two points A and B :



- We know that the centre of any circle passing through the two points A and B should be equidistant from A and B

∴ The centre of any circle passing through A and B should lie on the axis of symmetry of $\overline{AB}$ which is the straight line that is perpendicular to it from its midpoint , therefore , we draw the straight line L that represents the axis of symmetry of $\overline{AB}$



- We take a point (any point) on the straight line L as M , then we draw the circle whose centre is M and its radius length = MA or MB , then it will pass through the two points A and B
- Similarly we can draw another circle whose centre is $\grave{M}$ and its radius length = $\grave{M}A$ or $\grave{M}B$, then it will pass through the two points A and B or we can draw a circle whose centre is $\grave{M}$ and its radius length = $\grave{M}A$ or $\grave{M}B$, then it will pass through the two points A and B

There is an infinite number of circles that can be drawn to pass through the two points A and B and all their centres lie on the axis of symmetry of $\overline{AB}$

Third : Drawing a circle passing through three given points :

If A , B and C are three points in the plane and the required is drawing a circle passing through the three points A , B and C :

- We know that : In order that the circle can pass through the two points A and B , then , its centre should lie on the axis of symmetry of $\overline{AB}$, say L_1 and in order that the circle can pass through the two points B and C , its centre should lie on the axis of symmetry of $\overline{BC}$ say L_2

∴ The centre of the circle that passes through the three points A , B and C lies on each of L_1 and L_2

It is impossible to draw a circle passing through three collinear points.

For any three non-collinear points , there is a unique circle can be drawn to pass through them.

Notice that :

There is a unique circle passing through three points as A , B and C which are not collinear and the centre of this circle is the point of intersection of any two axes of symmetry of the axes of the line segments $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$

Corollary 1

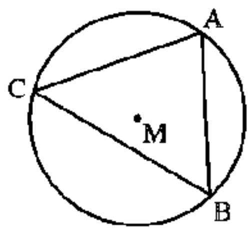
The circle which passes through the vertices of a triangle is called the circumcircle of this triangle.

- The triangle whose vertices lie on a circle is called the inscribed triangle of this circle.

In the opposite figure :

M is the circumcircle of Δ ABC

or Δ ABC is the inscribed triangle of the circle M



Corollary 2

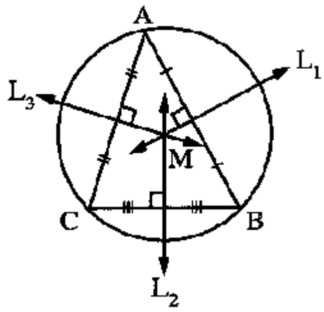
The perpendicular bisectors of the sides of a triangle intersect at a point which is the centre of the circumcircle of the triangle.

In the opposite figure :

If the straight lines L_1 , L_2 and L_3 are the axes of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively

and $L_1 \cap L_2 \cap L_3 = \{M\}$,

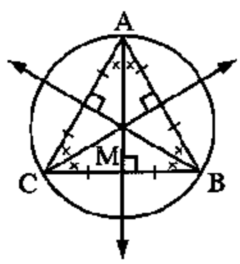
then the point M is the centre of the circumcircle of Δ ABC



- **A special case :**

The centre of the circumcircle of an equilateral triangle is :

- The point of intersection of its sides axes.
- The point of intersection of its altitudes.
- The point of intersection of its medians.
- The point of intersection of the bisectors of its angles.



Notice that :

We can draw a circle passing through the vertices of (a rectangle or a square or an isosceles trapezium) while we cannot draw a circle passing through the vertices of (the parallelogram or the rhombus or the trapezium which is not isosceles).

Choose the correct answer :

1 It is possible to draw .... passing through a given point. *(The New Valley 2005)*

(a) one circle (b) two circles

(c) three circles (d) an infinite number of circles

2 The number of circles passing through three collinear points is ....

(El-Sharkia 2013)

(a) zero (b) 1 (c) 2 (d) 3

3 We can identify the circle if we are given .... *(Sharkia 2008)*

(a) three collinear points. (b) two points.

(c) three non-collinear points. (d) one point.

4 The centre of the circumcircle of a triangle is the point of intersection of ....

(a) its medians. (b) its altitudes. *(Ismailia 2005)*

(c) the bisectors of its interior angles. (d) the axes of symmetry of its sides.

5 It is (impossible) to draw a circle passing through the vertices of ....

(El-Sharkia 2012 , El-Dakahlia 2013)

(a) a rectangle. (b) a triangle. (c) a square. (d) a rhombus.

Essay problems:

1 $\overline{AB}$ is of length 6 cm. Draw a circle passing through the two points A and B and its radius length is 4 cm. How many circles have you drawn ? *(Qena 2011)*

2 Draw a circle with radius length of 3 cm. and touches to the straight line L. What is the number of possible solutions ? *(Giza 2006)*

3 Draw ΔABC in which : $AB = 5$ cm. , $BC = 4$ cm. , and $CA = 3$ cm. What is the type of the triangle with respect to the measures of its angles ? then draw a circle whose centre is the point A and touches $\overleftrightarrow{BC}$, another circle whose centre is B and touches $\overleftrightarrow{AC}$ and a third circle whose centre is C and touches $\overleftrightarrow{AB}$ *(Beni Suef 2006)*

Homework

Choose the correct answer :

1 The number of circles which passes through two given points is ……… *(Giza 12)*

(a) 1 (b) 2
(c) 3 (d) an infinite number.

2 The number of circles passing through three collinear points is ………

(Giza 16 , Ismailia 15)

(a) zero (b) one (c) three (d) an infinite number.

3 The number of circles passing through three non-collinear points is ……… *(El-Menia 17)*

(a) 1 (b) zero (c) 2 (d) 3

4 The centres of the circles passing through the two points A and B lie on ………

(El-Dakahlia 17)

(a) the axis of symmetry of $\overline{AB}$ (b) $\overline{AB}$
(c) the perpendicular to $\overline{AB}$ (d) the midpoint of $\overline{AB}$

5 The centre of the circumcircle of a triangle is the point of intersection of ………

(Kafr El-Sheikh 17 , Qena 17)

(a) the bisectors of its interior angles. (b) the bisectors of its exterior angles.
(c) its altitudes. (d) the symmetry axes of its sides.

6 If ΔABC is right-angled at B , then the centre of its circumcircle is ……… *(Ismailia 03)*

(a) the midpoint of $\overline{AB}$ (b) the midpoint of $\overline{AC}$
(c) the midpoint of $\overline{BC}$ (d) outside the triangle.

7 It is possible to draw a circle passing through the vertices of ………

(Giza 17 , Beni Suef 16 , Qena 15)

(a) a rhombus. (b) a rectangle. (c) a trapezium. (d) a parallelogram.

8 If AB = 6 cm. , then the area of the smallest circle which passes through the two points A and B = ... cm^2. *(El-Sharkia 15)*

(a) 3π (b) 6π (c) 8π (d) 9π

Essay problems:

1 Using your geometric tools , draw $\overline{AB}$ of length 4 cm. , then draw on one figure :

(1) A circle passing through the two points A and B and its diameter length is 5 cm. What are the possible solutions ?

(2) A circle passing through the two points A and B and its radius length is 2 cm. What are the possible solutions ?

(3) A circle passing through the two points A and B and its diameter length is 3 cm. What are the possible solutions ?

Sheet (5)

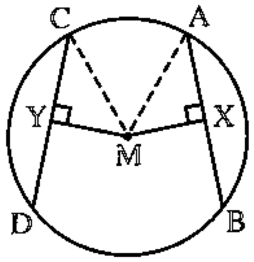
The relation between the chords of a circle and its centre

Theorem

If chords of a circle are equal in length, then they are equidistant from the centre.

$AB = CD$, $\overline{MX} \perp \overline{AB}$ and $\overline{MY} \perp \overline{CD}$

$MX = MY$



Corollary

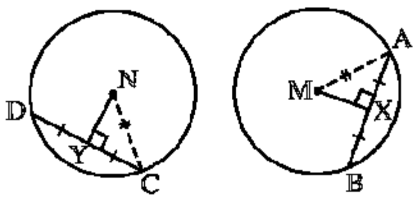
In congruent circles, chords which are equal in length are equidistant from the centres.

In the opposite figure :

If M and N are two congruent circles ,

$AB = CD$, $\overline{MX} \perp \overline{AB}$ and $\overline{NY} \perp \overline{CD}$,

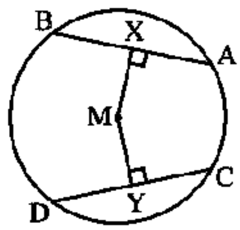
then $MX = NY$



Converse of the theorem

In the same circle (or in congruent circles) ,
chords which are equidistant from the centre (s) are equal in length.

i.e. **In the opposite figure :**



If $\overline{AB}$ and $\overline{CD}$ are two chords of the circle M ,

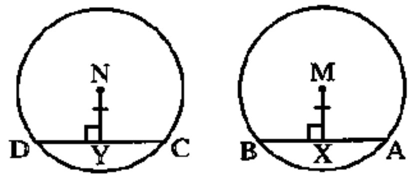
$\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{CD}$ and MX = MY , then AB = CD

Also in the opposite figure :

If M and N are two congruent circles , $\overline{AB}$ is a chord of circle M and $\overline{CD}$ is a chord of circle N

, $\overline{MX} \perp \overline{AB}$, $\overline{NY} \perp \overline{CD}$ and

MX = NY , then AB = CD



Complete:

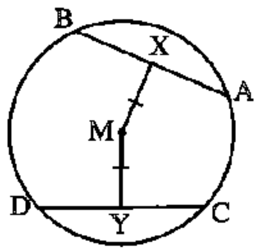
1 If the chords of a circle are equal in length , then they are .... from the ....

2 In the same circle if the chords are equidistant from the centre then they are ....

3 The square which is inscribed in a circle , its sides are .... from the centre of the circle. *(North Sinai 09)*

4 $\overline{AB}$ and $\overline{CD}$ are two chords in a circle AB = 5 cm. and CD = 3 cm. , then the chord which is nearer to the centre of the circle is ....

5 **In the opposite figure :**



If $\overline{AB}$ and $\overline{CD}$ are two chords in the circle M

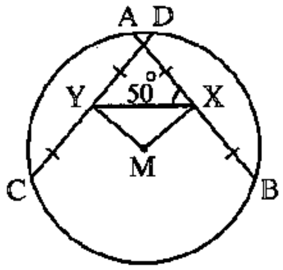
, X and Y are two midpoints of $\overline{AB}$

and $\overline{CD}$ respectively, if MX = MY , AB = 7 cm.

, then CY = ... cm. *(Red Sea 08)*

6 **In the opposite figure :**

$\overline{AB}$ and $\overline{CD}$ are two chords equal in length,
drawn in the circle M , X and Y are two
midpoints of $\overline{AB}$ and $\overline{CD}$ respectively.
If m (∠ AXY) = 50° , then m (∠ XMY) = ...°

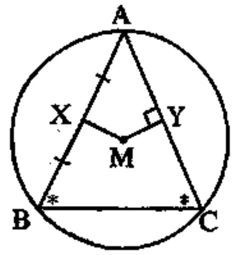


Essay problems:

1 **In the opposite figure :**

The triangle ABC is an inscribed triangle inside a circle M ,
m (∠ B) = m (∠ C) ,
X is the midpoint of $\overline{AB}$, $\overline{MY} \perp \overline{AC}$

Prove that : MX = MY

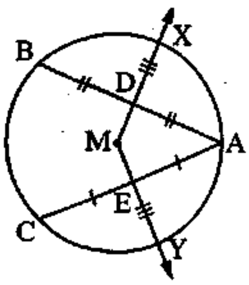


(Suez 2014 , Aswan 2011)

2 **In the opposite figure :**

M is a circle , $\overline{AB}$ and $\overline{AC}$ are two chords of it ,
D is the midpoint of $\overline{AB}$, E is the midpoint of
$\overline{AC}$, $\overrightarrow{MD}$ and $\overrightarrow{ME}$ are drawn to cut the circle
at X and Y respectively. If DX = EY

Prove that : AB = AC



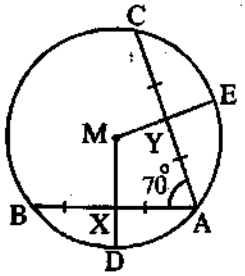
(Beheira 2008)

3 **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two chords equal in length in the circle M
, X is the midpoint of $\overline{AB}$,
Y is the midpoint of $\overline{AC}$ and m (∠ CAB) = 70°

1 **Calculate :** m (∠ DME)

2 **Prove that :** XD = YE

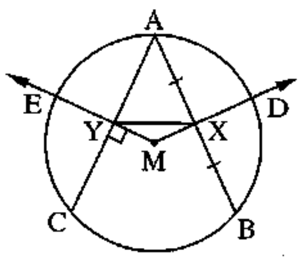


(Damietta 2013 , New Valley 2012)

4 **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two chords equal in length in the circle M
, X is the midpoint of $\overline{AB}$,
$\overrightarrow{MX}$ intersects the circle at D , $\overrightarrow{MY} \perp \overline{AC}$
intersects it at Y and intersects the circle at E

Prove that : **1** XD = YE **2** m (∠YXB) = m (∠ XYC)



(El-Gharbia 2013)

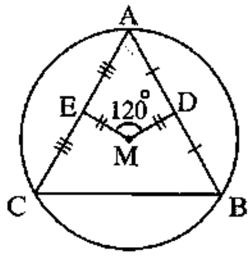
5 In the opposite figure :

Δ ABC is inscribed in the circle M ,

D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$

If DM = EM , m ($\angle$ DME) = 120°

Prove that : Δ ABC is an equilateral triangle.



(Menia 2003)

Homework

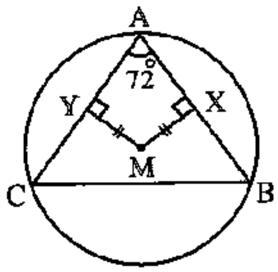
Complete:

1 **In the opposite figure :**

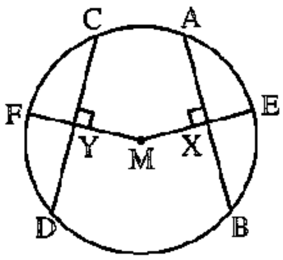
Δ ABC is inscribed in the circle M ,

$\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{AC}$, MX = MY

and m ($\angle$ A) = 72° , then m ($\angle$ B) = ………



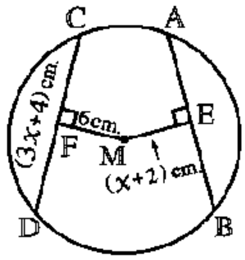
2 (1)



If AB = CD , then MX = ………

$\because$ ME = ……… $\therefore$ EX = ………

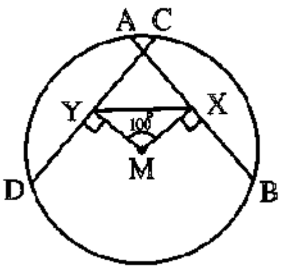
(2)



If AB = CD , then ME = ………

$\therefore x$ = ……… cm. $\therefore$ CD = ……… cm.

3 (3)



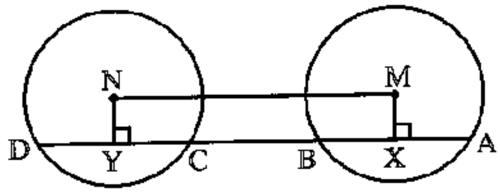
If AB = CD , then MX = ………

In Δ MXY

$\because$ m ($\angle$ XMY) = 100°

$\therefore$ m ($\angle$ MXY) = ……… °

(4)



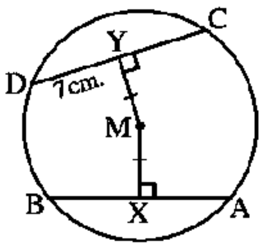
If M and N are two congruent circles ,

AB = CD , then MX = ………

and the figure MXYN is ………

4

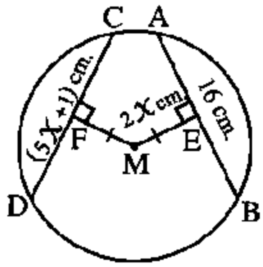
(1)



If MX = MY , YD = 7 cm.

, then AB = ………. cm.

(2)



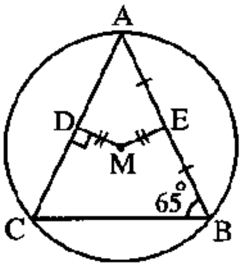
If ME = MF , then CD = ……….

∴ X = ………. cm. , EM = ………. cm.

AM = ………. cm.

5

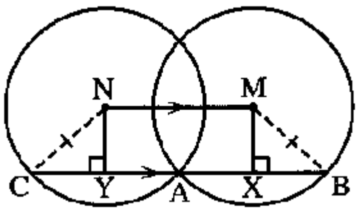
(3)



If MD = ME , m (∠ B) = 65°

, then m (∠ A) = ……….°

(4)



∵ $\overleftrightarrow{MN}$ // $\overleftrightarrow{BC}$ ∴ MX = ……….

∵ The two circles M and N are ………. , A ∈ $\overline{BC}$

∴ AB = ……….

Essay problems:

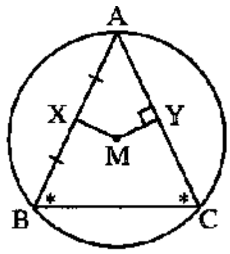
1

In the opposite figure :

The triangle ABC is an inscribed triangle inside a circle M ,

m (∠ B) = m (∠ C) ,

X is the midpoint of $\overline{AB}$, $\overrightarrow{MY} \perp \overline{AC}$



Prove that : MX = MY

(Matrouh 17 , Fayoum 15 , Suez 14 , Aswan 11)

2

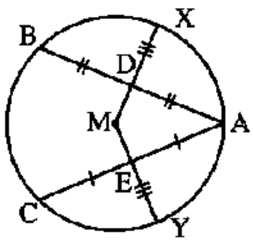
In the opposite figure :

$\overline{AB}$ and $\overline{AC}$ are two chords in the circle M

, D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

, DX = EY



Prove that : AB = AC

(El-Kalyoubia 16)

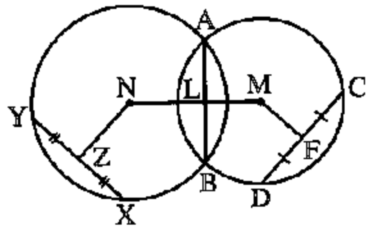
3 **In the opposite figure :**

M and N are two circles intersecting at A and B ,

$\overline{MN} \cap \overline{AB} = \{L\}$, F is the midpoint of $\overline{CD}$,

Z is the midpoint of $\overline{XY}$, MF = ML and NL = NZ

Prove that : CD = XY

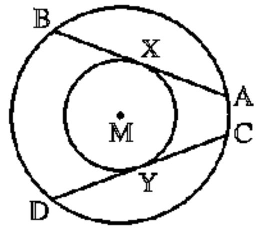


(Monofia 09)

4 **In the opposite figure :**

Two concentric circles at M , $\overline{AB}$ and $\overline{CD}$ are two chords of the greater circle and touch the smaller circle at X and Y respectively.

Prove that : AB = CD if the radius length of the greater circle = 5 cm. and the radius length of the smaller circle = 3 cm. , find the length of $\overline{AB}$



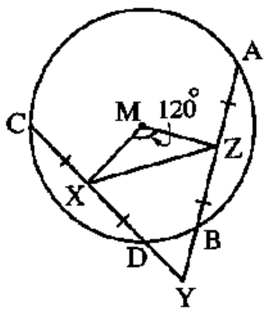
(Gharbia 04) « 8 cm. »

5 **In the opposite figure :**

$\overline{AB}$ and $\overline{CD}$ are two chords of the circle M equal in length , $\overrightarrow{AB} \cap \overrightarrow{CD} = \{Y\}$,

Z is the midpoint of $\overline{AB}$, X is the midpoint of $\overline{CD}$ and m (∠ ZMX) = 120°

Prove that : Δ ZYX is an equilateral triangle.



6 **In the opposite figure :**

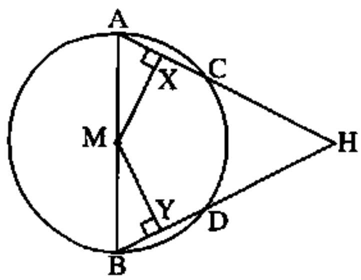
$\overline{AB}$ is a diameter of the circle M , $\overline{AC}$ and $\overline{BD}$ are two chords in it ,

MX = MY , $\overline{MX} \perp \overline{AC}$, $\overline{MY} \perp \overline{DB}$

Prove that :

1 Δ HAB is isosceles triangle.

2 HC = HD



(Beni-Suef 2012)

Sheet (6)

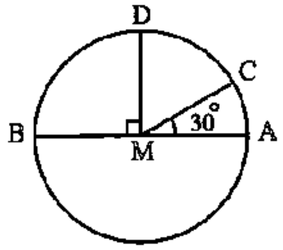
The relation between the chords of a circle and its centre

The measure of the arc:

It is the measure of the central angle which subtends this arc and it is measured by the measuring units of the angle (degrees , minutes , seconds ...)

For example:

In the opposite figure :



If $\overline{AB}$ is a diameter of the circle M , C and D are two points on the circle M where m (∠ AMC) = 30° , m (∠ AMD) = 90° , then :

1. m $(\overset{\frown}{AC})$ = m (∠ AMC) = 30°
2. m $(\overset{\frown}{CD})$ = m (∠ CMD) = 90° – 30° = 60°
3. m $(\overset{\frown}{DB})$ = m (∠ DMB) = 90°
4. m $(\overset{\frown}{DB}$ the major) = m (∠ DMB the reflex) = 360° – 90° = 270°
5. m $(\overset{\frown}{AB})$ = m (∠ AMB) = 180° (Notice that : $\overset{\frown}{AB}$ represents a semicircle)

The length of the arc:

It is part of a circle's circumference proportional to its measure and it is measured by length units (centimetre , metre , ...)

To calculate the length of the arc, you can use the following rule :

$$\text{The length of the arc} = \frac{\text{the measure of the arc}}{\text{the measure of the circle}} \times \text{the circumference of the circle}$$

$$= \frac{\text{the measure of the arc}}{360°} \times 2\pi r$$

Where r is the radius length of the circle and π is the approximated ratio.

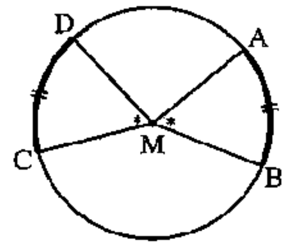
Remark

The length of the semicircle = $\frac{1}{2}$ the circumference of the circle = πr length unit

Corollary (1):

In the same circle (or in congruent circles) , if the measures of arcs are equal , then the lengths of the arcs are equal and vice versa.

In the opposite figure :



If M is a circle in which m $(\overset{\frown}{AB})$ = m $(\overset{\frown}{CD})$

, then the length of $\overset{\frown}{AB}$ = the length of $\overset{\frown}{CD}$

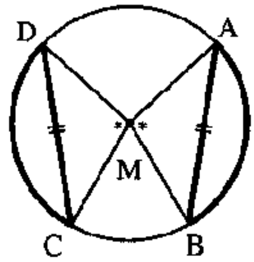
and vice versa : If the length of $\overset{\frown}{AB}$ = the length of $\overset{\frown}{CD}$

, then m $(\overset{\frown}{AB})$ = m $(\overset{\frown}{CD})$

Corollary (2):

In the same circle (or in congruent circles) , if the measures of arcs are equal , then their chords are equal in length , and vice versa.

In the opposite figure :



If M is a circle in which

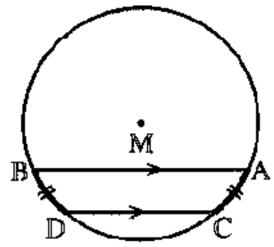
m $(\overset{\frown}{AB})$ = m $(\overset{\frown}{CD})$, then AB = CD

and vice versa : If AB = CD , then m $(\overset{\frown}{AB})$ = m $(\overset{\frown}{CD})$

Corollary (3):

If two parallel chords are drawn in a circle , then the measures of the two arcs between them are equal.

In the opposite figure :



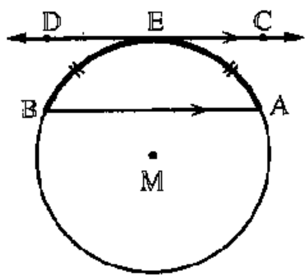
If $\overline{AB}$ and $\overline{CD}$ are two chords in the circle M

, $\overline{AB}$ // $\overline{CD}$, then m $(\overset{\frown}{AC})$ = m $(\overset{\frown}{BD})$

Corollary (4):

If a chord is parallel to a tangent of a circle , then the measures of the two arcs between them are equal.

In the opposite figure :



If $\overline{AB}$ is a chord in the circle M and

$\overleftrightarrow{CD}$ touches the circle M at E ,

$\overleftrightarrow{CD}$ // $\overline{AB}$, then m $(\overset{\frown}{EA})$ = m $(\overset{\frown}{EB})$

Choose the correct answer :

1 The central angle whose measure is 90° subtends an arc of length = ……… the circumference of the circle. *(Assiut 11)*

(a) $\frac{1}{4}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$

2 The circumference of a circle = 36 cm., then the measure of an arc of it with length = 6 cm. is ………

(a) 60° (b) 30° (c) 90° (d) 120°

3 The length of the arc opposite to a central angle whose measure = 120° in a circle of radius length r equals ……… *(Suez 09)*

(a) $\frac{1}{3}\pi r$ (b) πr (c) $\frac{2}{3}\pi r$ (d) $3\pi r$

4 The length of the arc which represents $\frac{1}{4}$ the circumference of the circle = ……… cm.

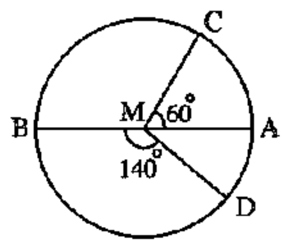
(a) $2\pi r$ (b) πr (c) $\frac{1}{2}\pi r$ (d) $4\pi r$

5 The measure of the arc which represents $\frac{1}{6}$ the circumference of the circle = ……… *(Cairo 15)*

(a) 60° (b) 90° (c) 120° (d) 300°

Complete:

1 **In the opposite figure :**



$\overline{AB}$ is a diameter of the circle M , m (∠ AMC) = 60° ,

m (∠ BMD) = 140°

Complete the following :

(1) m $(\overset{\frown}{AC})$ = ………°

(2) m $(\overset{\frown}{BD})$ = ………°

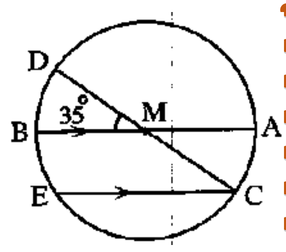
(3) m $(\overset{\frown}{CD})$ = ………°

(4) m $(\overset{\frown}{DBC})$ = ………°

(5) m $(\overset{\frown}{DCB})$ = ………°

Essay problems:

1 **In the opposite figure :**



$\overline{AB}$ and $\overline{CD}$ are two diameters in the circle M

such that : m (∠ DMB) = 35° , $\overline{CE}$ // $\overline{AB}$

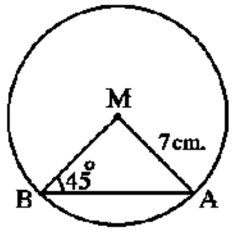
Find : m $(\overset{\frown}{BE})$

2 **In the opposite figure :**

A and B are two points belonging to the circle M

such that : m (∠ MBA) = 45° , AM = 7 cm.

Find : The length of $\overset{\frown}{AB}$ ($\pi = \frac{22}{7}$)

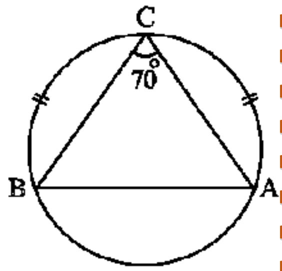


3 **In the opposite figure :**

If m ($\overset{\frown}{AC}$) = m ($\overset{\frown}{BC}$)

, m (∠ ACB) = 70°

Find : m (∠ ABC)

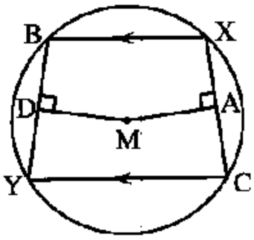


4 **In the opposite figure :**

$\overline{XB} \parallel \overline{CY}$, $\overline{MA} \perp \overline{XC}$, $\overline{MD} \perp \overline{BY}$

Prove that : MA = MD

(Giza 17)



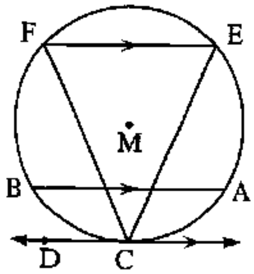
5 📖 **In the opposite figure :**

M is a circle , $\overleftrightarrow{CD}$ is a tangent to the circle at C ,

$\overline{AB}$ and $\overline{EF}$ are two chords in the circle

, where $\overline{AB} \parallel \overline{EF} \parallel \overleftrightarrow{CD}$

Prove that : CE = CF



(El-Beheira 2014 , Alex. 2011)

6 ABCD is a quadrilateral inscribed in the circle M such that AB = CD. **Prove that :** AC = BD

Homework

Choose the correct answer :

1 The length of the arc opposite to a central angle of measure 30° in a circle of circumference 36 cm. = ... cm. *(Souhag 09)*

(a) 18 (b) 9 (c) 3 (d) 4.5

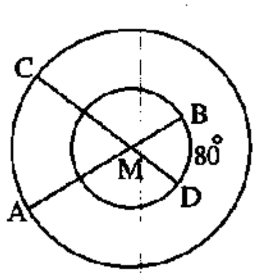
2 An arc in a circle , its length = $\frac{1}{3}$ πr , then it is opposite to a central angle of measure .... *(Beni Suef 16 , El-Menia 13 , Kafr El-Sheikh 15)*

(a) 30° (b) 60° (c) 120° (d) 240°

3 If A and B are two points belonging to a circle M such that the length of $\overset{\frown}{AB} = \pi r$, then $\overline{AB}$ is ……… in the circle M

(a) a radius
(b) a chord not passing through the centre
(c) a diameter
(d) an axis of symmetry of the circle

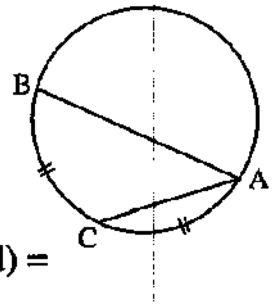
4 **In the opposite figure :**



Two concentric circles with centre M, $\overline{AB} \cap \overline{CD} = \{M\}$, if $m(\overset{\frown}{BD}) = 80°$, then $m(\overset{\frown}{AC}) = ………$

(a) 40°
(b) 60°
(c) 80°
(d) 160°

5 **In the opposite figure :**



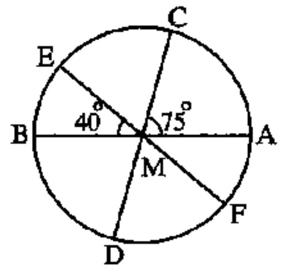
If C is the midpoint of $\overset{\frown}{AB}$, then AB ……… 2 AC

(Giza 17)

(a) <
(b) >
(c) ≥
(d) =

Complete:

1 **In the opposite figure :**

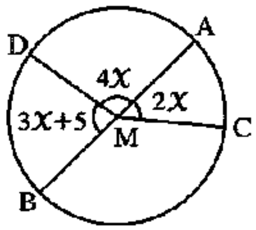


$\overline{AB}$, $\overline{CD}$ and $\overline{EF}$ are diameters of the circle M

Complete :

(1) $m(\overset{\frown}{AC}) = ………°$
(2) $m(\overset{\frown}{ACE}) = ………°$
(3) $m(\overset{\frown}{ACD}) = ………°$
(4) $m(\overset{\frown}{AFE}) = ………°$

2 **In the opposite figure :**



$\overline{AB}$ is a diameter of the circle M, study the figure, then complete :

(1) $X = ………°$
(2) $m(\overset{\frown}{AC}) = ………°$
(3) $m(\overset{\frown}{AD}) = ………°$
(4) $m(\overset{\frown}{BC}) = ………°$
(5) $m(\overset{\frown}{CAD}) = ………°$
(6) $m(\overset{\frown}{CBD}) = ………°$
(7) $m(\overset{\frown}{ACD}) = ………°$
(8) $m(\overset{\frown}{ADC}) = ………°$

Essay problems:

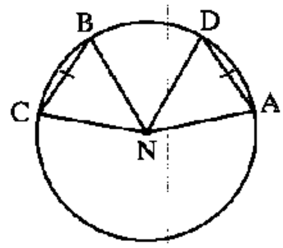
1 **In the opposite figure :**

A and B are two points belonging to the circle N

, D $\in \overset{\frown}{AB}$, C $\in$ the major arc $\overset{\frown}{AB}$

such that AD = BC

Prove that : m ($\angle$ ANB) = m ($\angle$ CND)



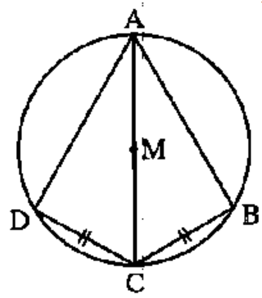
(Souhag 05)

2 **In the opposite figure :**

ABCD is a quadrilateral inscribed in a circle M

, $\overline{AC}$ is a diameter in the circle , CB = CD

Prove that : m ($\overset{\frown}{AB}$) = m ($\overset{\frown}{AD}$)



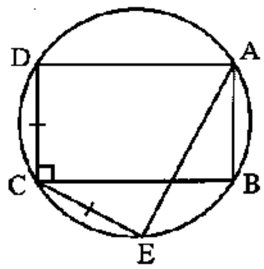
3 **In the opposite figure :**

ABCD is a rectangle inscribed

in a circle. Draw the chord $\overline{CE}$

, where CE = CD

Prove that : AE = BC

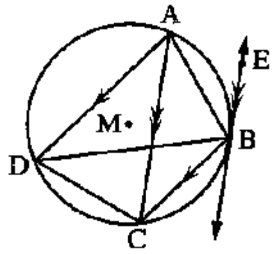


4 **In the opposite figure :**

$\overleftrightarrow{BE}$ is a tangent to the circle M at B

, $\overline{BC}$ // $\overline{AD}$, $\overleftrightarrow{BE}$ // $\overline{AC}$

Prove that : Δ BCD is isosceles.



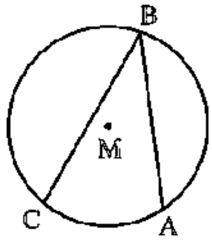
Sheet (7)

The relation between the inscribed and central angles subtended by the same arc (theorem 1)

The inscribed angle:

It is the angle whose vertex lies on the circle and its sides contain two chords of the circle.

In the opposite figure :



- $\angle ABC$ is an inscribed angle

  because its vertex B belongs to the circle M

  and its sides $\overrightarrow{BA}$ and $\overrightarrow{BC}$ carry the two chords $\overline{BA}$ and $\overline{BC}$ in the circle M

- The inscribed angle $\angle ABC$ is subtended by $\overset{\frown}{AC}$

Remark

For each inscribed angle , there is one central angle subtended by the same arc.

Theorem 1

The measure of the inscribed angle is half the measure of the central angle , subtended by the same arc.

Remark

The measure of the central angle equals twice the measure of the inscribed angle subtended by the same arc.

In each of the following figures, find the measure of each angle denoted by (?) given that M is the centre of the circle :

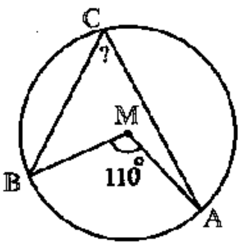


Fig. (1)

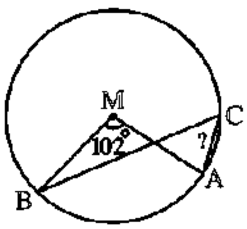


Fig. (2)

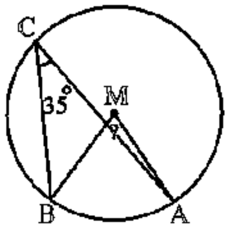


Fig. (3)

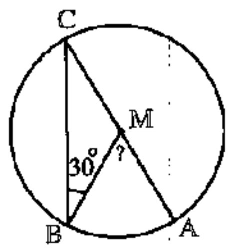


Fig. (4)

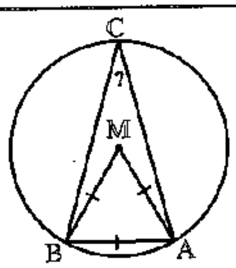


Fig. (5)

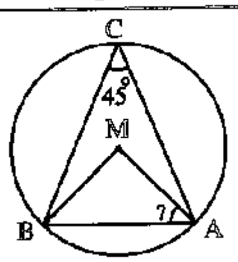


Fig. (6)

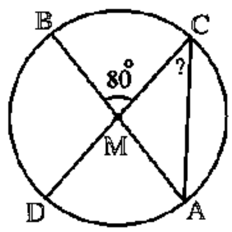


Fig. (7)

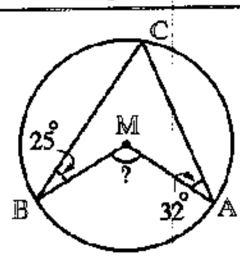


Fig. (8)

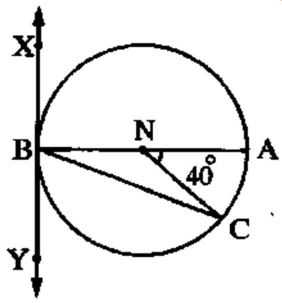
Choose the correct answer :

1 The ratio between the measure of the central angle and the measure of the inscribed angle that has the same subtended arc = ...

(a) 3 : 1 (b) 2 : 1 (c) 1 : 2 (d) 1 : 3

(El-Fayoum 2011)

2 **In the opposite figure :**



$\overline{AB}$ is a diameter in the circle N

, $\overleftrightarrow{XY}$ is a tangent to the circle at B

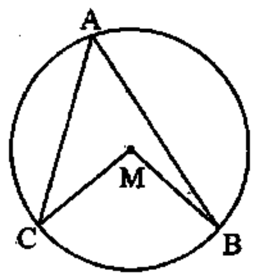
, m (∠ ANC) = 40° , then m (∠ CBY) = ...

(a) 40° (b) 50°

(c) 20° (d) 70°

(El-Fayoum 2006)

3 **In the opposite figure :**



M is a circle , m (∠ M) – m (∠ A) = 50°

, then m (∠ A) = ...

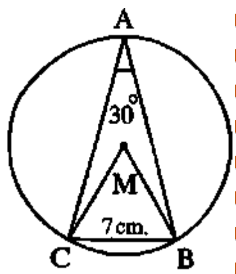
(a) 40° (b) 50°

(c) 100° (d) 130°

(Port Said 2013)

Essay problems:

1 **In the opposite figure :**



m (∠ A) = 30° , BC = 7 cm.

Find : The area of the circle M ($\pi = \frac{22}{7}$)

2 **Using the opposite figure :**

Write the given data then find : **1** m (∠ ABC)

2 m ($\overset{\frown}{BC}$ the major)

(New Valley 2006) « 70° , 280° »

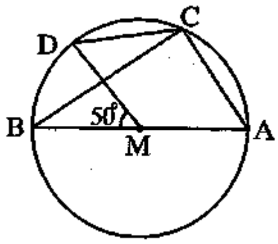
3 **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M ,

m (∠ BMD) = 50°

Find with proof :

m (∠ ACD)



(*Damietta 2014*) « 115° »

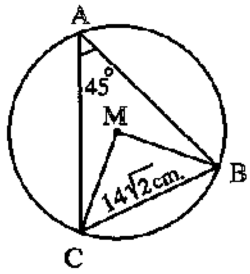
4 **In the opposite figure :**

M is a circle ,

m (∠ A) = 45°

and BC = $14\sqrt{2}$ cm.

Find : The area of the circle M $\left(\pi = \frac{22}{7}\right)$



« 616 cm^2 »

Homework

In each of the following figures, find the measure of each angle denoted by (?) given that M is the centre of the circle :

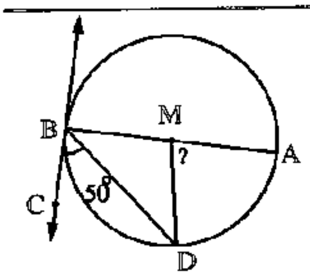


Fig. (9)

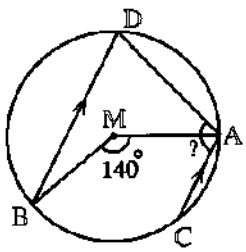


Fig. (10)

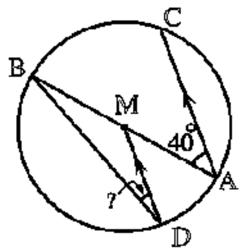


Fig. (11)

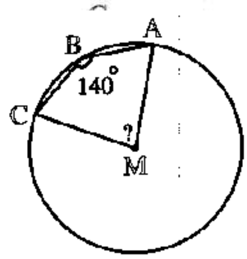


Fig. (12)

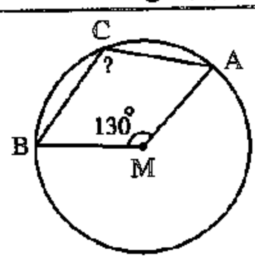


Fig. (13)

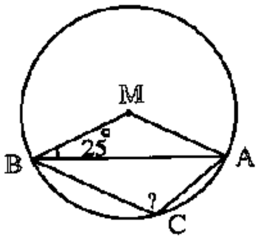


Fig. (14)

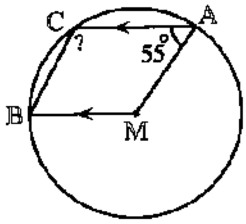


Fig. (15)

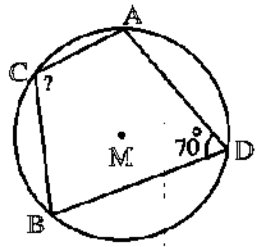


Fig. (16)

Choose the correct answer :

1 The measure of the inscribed angle equals .... the measure of the central angle subtended by the same arc. (*El-Menia 2014*)

(a) half (b) twice (c) quarter (d) third

2 If the measure of a central angle is 100° , then the measure of the inscribed angle that has the same subtended arc = ... (*Giza 2011*)

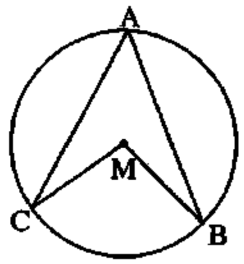
(a) 200° (b) 100° (c) 50° (d) 25°

3 **In the opposite figure :**

M is a circle

, m (∠ A) + m (∠ BMC) = 150°

, then m (∠ A) = ……….



(a) 100° (b) 45°

(c) 75° (d) 50°

(Helwan 2009)

4 The inscribed angle which is subtended by an arc of measure greater than the measure of a semicircle is ……… angle. *(Qena 2011)*

(a) a straight (b) an acute (c) a right (d) an obtuse

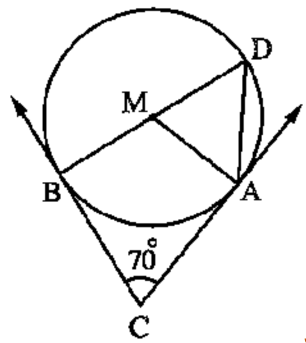
5 The ratio between the measure of the central angle and the measure of the inscribed angle that has the same subtended arc = ……… *(El-Fayoum 11)*

(a) 3 : 1 (b) 2 : 1 (c) 1 : 2 (d) 1 : 3

6 **In the opposite figure :**

If $\overrightarrow{CA}$, $\overrightarrow{CB}$ are two tangents to the circle M , m (∠ C) = 70°

, then m (∠ ADM) = ………



(a) 35° (b) 55°

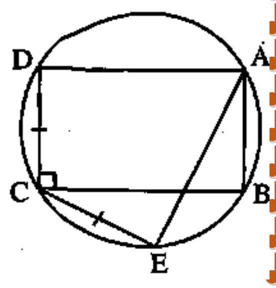
(c) 65° (d) 45°

Essay problems:

1 **In the opposite figure :**

ABCD is a rectangle inscribed in a circle. Draw the chord $\overline{CE}$, where CE = CD

Prove that : AE = BC



2 ABCD is a quadrilateral inscribed in a circle. If $\overline{AB}$ // $\overline{DC}$, E is the midpoint of $\overset{\frown}{AB}$

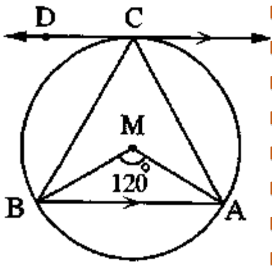
Prove that : CE = DE *(Giza 2005)*

3 📖 **In the opposite figure :**

$\overleftrightarrow{CD}$ is a tangent to the circle at C

, $\overleftrightarrow{CD}$ // $\overline{AB}$, m (∠ AMB) = 120°

Prove that : Δ CAB is equilateral. *(Ismailia 2013)*



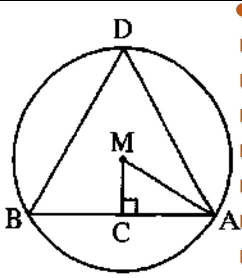
4 📖 **In the opposite figure :**

$\overline{AB}$ is a chord in the circle M ,

$\overline{MC}$ ⊥ $\overline{AB}$

Prove that : m (∠ AMC) = m (∠ ADB)

(Port Said 2014 , El-Beheira 2013)

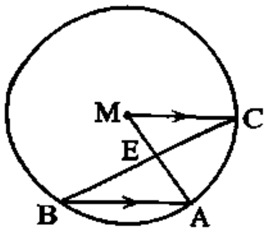


5 📖 **In the opposite figure :**

$\overline{AB}$ is a chord in the circle M ,

$\overline{CM}$ // $\overline{AB}$, $\overline{BC}$ ∩ $\overline{AM}$ = {E}

Prove that : BE > AE *(El-Gharbia 2014 , Fayoum 2012 , El-Dakahlia 2013)*

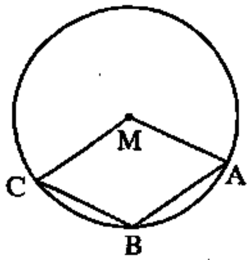


6 **In the opposite figure :**

If M is the centre of the circle

, m (∠ AMC) = m (∠ B)

Find : m (∠ B)

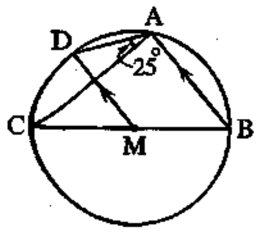


7 **In the opposite figure :**

$\overline{BC}$ is a diameter in the circle M ,

$\overline{MD}$ // $\overline{BA}$, m (∠ CAD) = 25°

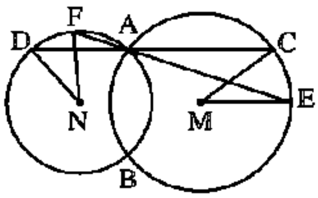
Find : m (∠ ACB) *(Sharkia , Dakahlia 2012)* « 40° »



8 **In the opposite figure :**

M and N are two circles intersecting in A and B and the two straight lines $\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ pass through the point A and intersect the circle M at C and E and the circle N at D and F

Prove that : m (∠ CME) = m (∠ FND)



Sheet (8)
Corollaries of theorem (1) and its well known peroblems

Corollary (1):

The measure of an inscribed angle is half the measure of the subtended arc.

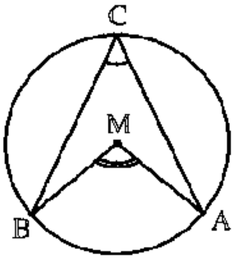
In the opposite figure :

$m(\angle C) = \frac{1}{2} m(\angle AMB)$

(inscribed and central angles with common arc $\overset{\frown}{AB}$),

$m(\angle AMB) = m(\overset{\frown}{AB})$

$\therefore m(\angle C) = \frac{1}{2} m(\overset{\frown}{AB})$



Remark

The measure of the arc equals twice the measure of the inscribed angle subtended by this arc.

Corollary (2):

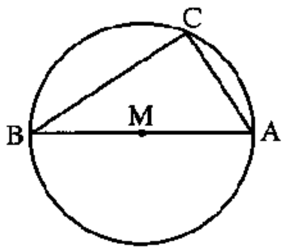
The inscribed angle in a semicircle is a right angle.

In the opposite figure :

$\because m(\angle C) = \frac{1}{2} m(\overset{\frown}{AB})$ (corollary 1)

$\therefore m(\overset{\frown}{AB}) = 180°$

$\therefore m(\angle C) = 90°$



Remarks

1. The inscribed angle which is right angle is drawn in a semicircle.
2. The inscribed angle which is subtended by an arc of measure less than the measure of a semicircle is an acute angle.
3. The inscribed angle which is subtended by an arc of measure greater than the measure of a semicircle is an obtuse angle.

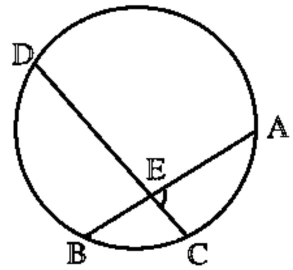
Well known problem (1):

If two chords intersect at a point inside a circle , then the measure of the included angle equals half of the sum of the two measures of the two opposite arcs.

$\overline{AB}$, $\overline{CD}$ are two chords in a circle intersecting at the point E

$$m(\angle AEC) = \frac{1}{2}\left[m(\overset{\frown}{AC}) + m(\overset{\frown}{BD})\right]$$

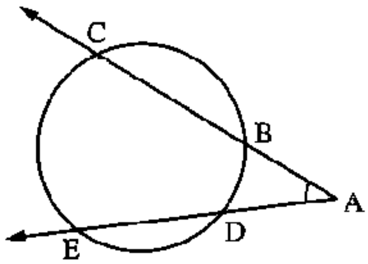
$$m(\angle CEB) = \frac{1}{2}\left[m(\overset{\frown}{BC}) + m(\overset{\frown}{AD})\right]$$



Well known problem (2):

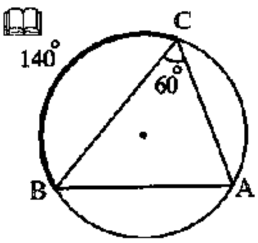
If two rays carrying two chords in a circle are intersecting outside it, then the measure of their intersecting angle equals half of the measure of the major arc subtracted from it half of the measure of the minor arc in which both are included by the two sides of this angle.

$$m(\angle A) = \frac{1}{2}\left[m(\overset{\frown}{CE}) - m(\overset{\frown}{BD})\right]$$



Study each of the following figures , then complete :

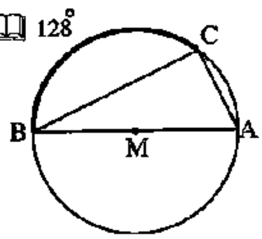
(1)



$m(\angle A) = \dots\dots\dots^\circ$

$m(\overset{\frown}{AC}) = \dots\dots\dots^\circ$

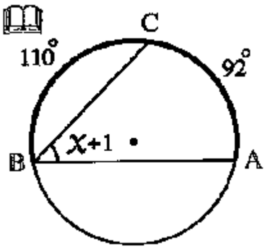
(2)



$m(\angle C) = \dots\dots\dots^\circ$

$m(\angle B) = \dots\dots\dots^\circ$

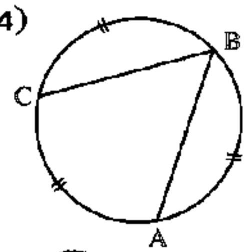
(3)



$X = \dots\dots\dots^\circ$

$m(\overset{\frown}{AB}) = \dots\dots\dots^\circ$

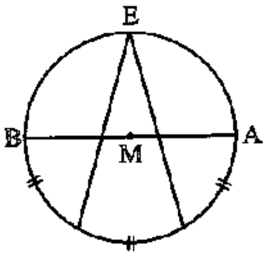
(4)



$m(\overset{\frown}{AC}) = \dots\dots\dots^\circ$

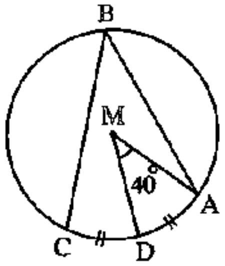
$m(\angle B) = \dots\dots\dots^\circ$

(5)



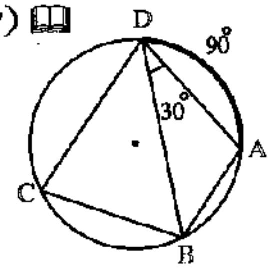
$m(\angle E) = \dots\dots\dots^\circ$

(6)



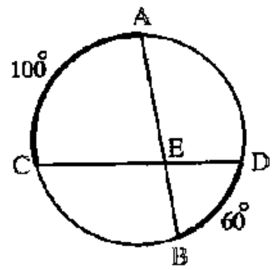
$m(\angle B) = \dots\dots\dots^\circ$

(7)



$m(\angle DCB) = \dots\dots\dots^\circ$

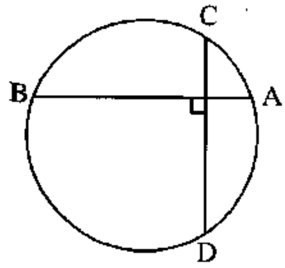
(8)



$m(\angle AEC) = \dots\dots\dots^\circ$

Choose the correct answer :

1 **In the opposite figure :**

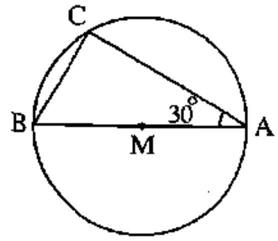


m $(\overset{\frown}{AC})$ + m $(\overset{\frown}{BD})$ = ...

(a) 45° (b) 90°

(c) 180° (d) 270°

2 **In the opposite figure :**



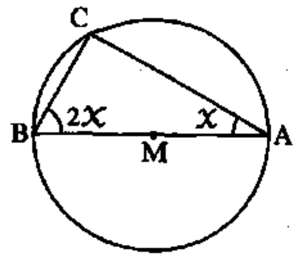
$\overline{AB}$ is a diameter in the circle M of radius length 4 cm. , m (∠ A) = 30° , then BC = ... cm.

(a) 2 (b) 4

(c) 6 (d) 8

(Matrouh 2011)

3 **In the opposite figure :**



If $\overline{AB}$ is a diameter in the circle M ,

then x = ...

(a) 40° (b) 20°

(c) 30° (d) 60°

(Menia 2012)

4 The inscribed angle which is subtended by minor arc in a circle is ....

(Alex. 17 , Qena 16)

(a) reflex. (b) right. (c) obtuse. (d) acute.

5 The length of the arc that is opposite a right inscribed angle in a circle whose circumference is 44 cm. equals .... cm.

(Dakahlia 12)

(a) 22 (b) 11 (c) $\frac{22}{7}$ (d) $\frac{44}{7}$

6 The measure of the inscribed angle which is drawn in $\frac{1}{3}$ a circle equals ....

(Cairo 09)

(a) 240° (b) 120° (c) 60° (d) 30°

7 The measure of the inscribed angle which is subtended by an arc representing $\frac{1}{3}$ a circle equals ....

(a) 240° (b) 120° (c) 60° (d) 30°

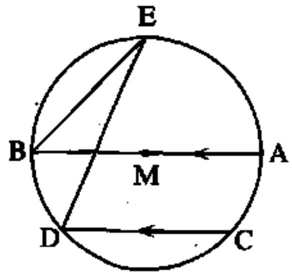
Essay problems:

1 **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M ,

$\overline{AB} \parallel \overline{CD}$ and m $(\overset{\frown}{CD}) = 80°$

Find : m (∠ E)



(Souhag 2012) « 25° »

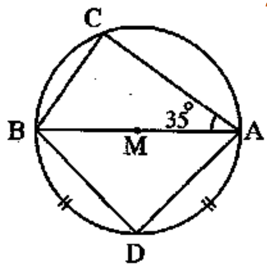
2 **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M ,

the length of $\overset{\frown}{AD}$ = the length of $\overset{\frown}{BD}$,

m (∠ CAB) = 35°

Find by proof : m (∠ CBD)



(Menia 2011) « 100° »

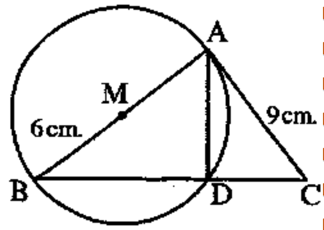
3 **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, $\overline{AC}$ touches the circle at A

If AC = 9 cm. , BM = 6 cm.

Find the length of each of : $\overline{BC}$, $\overline{AD}$



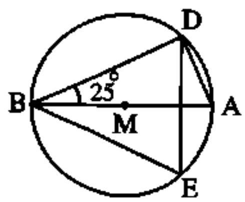
(Kafr El-Sheikh 2004) « 15 cm., 7.2 cm. »

4 **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, m (∠ ABD) = 25°

Find : m (∠ DEB) in degrees.



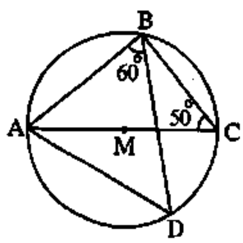
(Suez 2011) « 65° »

5 **In the opposite figure :**

$\overline{AC}$ is a diameter in the circle M

, m (∠ C) = 50° , m (∠ ABD) = 60°

Find with proof : m (∠ CBD) and m (∠ BAD)



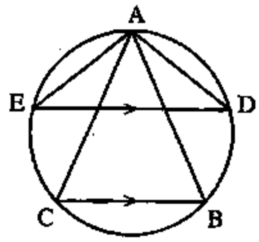
(Kafr El-Sheikh 2013) « 30° , 70° »

6 **In the opposite figure :**

ABC is a triangle inscribed in a circle ,

$\overline{DE} \parallel \overline{BC}$

Prove that : m (∠ DAC) = m (∠ BAE)

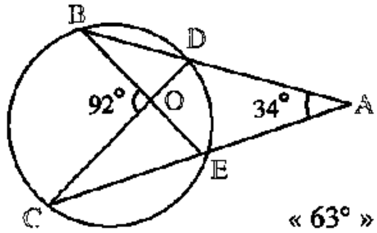


7 **In the opposite figure :**

m (∠ A) = 34°

, m (∠ BOC) = 92°

Find : m (∠ CDB)



« 63° »

Homework

Choose the correct answer :

1 The inscribed angle which is subtended by an arc of measure greater than the measure of a semicircle is .... angle. *(Qena 2011)*

(a) a straight (b) an acute (c) a right (d) an obtuse

2 The measure of the inscribed angle which is drawn in $\frac{1}{3}$ a circle equals ....

(Cairo 2009)

(a) 240° (b) 120° (c) 60° (d) 30°

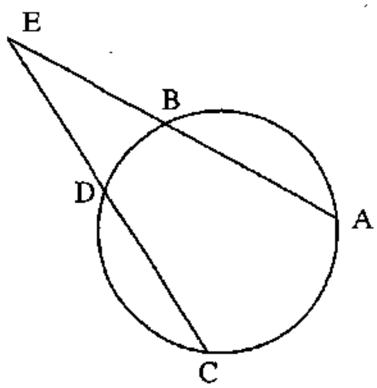
3 **In the opposite figure :**

If m $(\overset{\frown}{AC})$ − m $(\overset{\frown}{BD})$ = 70°

, then m (∠ E) = ...

(a) 35° (b) 70°

(c) 110° (d) 140°



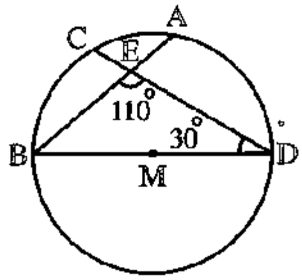
4 **In the opposite figure :**

$\overline{AB} \cap \overline{CD} = \{E\}$, m (∠ D) = 30° , m (∠ DEB) = 110° ,

then m $(\overset{\frown}{AD})$ = ...

(a) 80° (b) 70°

(c) 40° (d) 60°



(Kalyoubia 05)

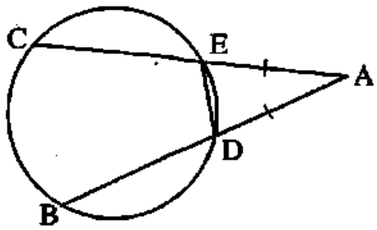
5 **In the opposite figure :**

If m $(\overset{\frown}{BC}) = 112°$, m $(\overset{\frown}{DE}) = 44°$, AD = AE ,

then m $(\angle ADE) =$ ....

(a) 75° (b) 73°

(c) 70° (d) 76°



(Kafr El-Sheikh 2008)

Essay problems:

1 ABCD is a quadrilateral inscribed in a circle. If $\overline{AB} \mathbin{/\mkern-6mu/} \overline{DC}$, E is the midpoint of $\overset{\frown}{AB}$

Prove that : CE = DE

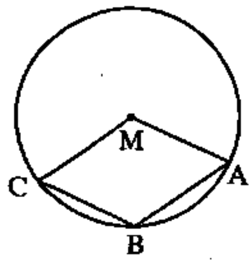
(Giza 2005)

2 **In the opposite figure :**

If M is the centre of the circle

, m $(\angle AMC) = $ m $(\angle B)$

Find : m $(\angle B)$



(Monofia 2006) « 120° »

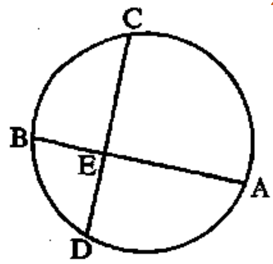
3 **In the opposite figure :**

$\overline{AB}$ and $\overline{CD}$ are two chords in the circle ,

$\overline{AB} \cap \overline{CD} = \{E\}$, if m $(\overset{\frown}{BD}) = 60°$, m $(\overset{\frown}{AD}) = 100°$,

m $(\overset{\frown}{AC}) = 120°$

Calculate : 1 m $(\overset{\frown}{CB})$ 2 m $(\angle CEB)$



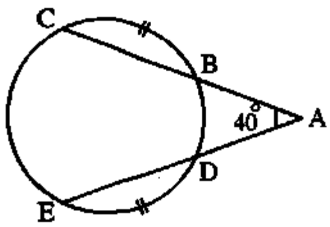
(Alex. 2005) « 80° , 90° »

4 **In the opposite figure :**

m $(\angle A) = 40°$, m $(\overset{\frown}{BD}) = 60°$

, m $(\overset{\frown}{BC}) = $ m $(\overset{\frown}{DE})$

Find : 1 m $(\overset{\frown}{EC})$ 2 m $(\overset{\frown}{BC})$



(El-Monofia 2005) « 140° , 80° »

5

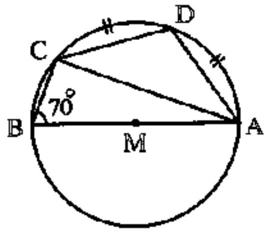
In the opposite figure :

$\overline{AB}$ is a diameter in the circle M ,

the length of $(\overset{\frown}{AD})$ = the length of $(\overset{\frown}{DC})$,

m (∠ ABC) = 70°

Find each of : m (∠ DCA) , m (∠ CAB)



(El-Ismailia 05) « 35° , 20° »

6

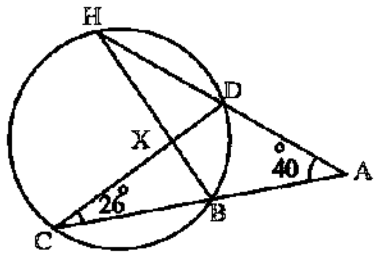
In the opposite figure :

$\overrightarrow{CB} \cap \overrightarrow{HD} = \{A\}$, m (∠ A) = 40°

$\overline{DC} \cap \overline{BH} = \{X\}$ and m (∠ DCB) = 26°

Find :

(1) m $(\overset{\frown}{CH})$ (2) m (∠ HXC)



(El-Gharbia 17 , Ismailia 16) « 132° , 92° »

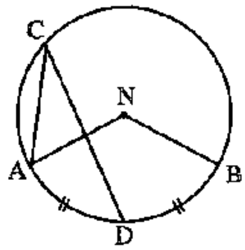
7

In the opposite figure :

D is the midpoint of $\overset{\frown}{AB}$

Prove that :

m (∠ ACD) = $\frac{1}{4}$ m (∠ ANB)



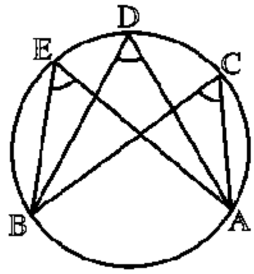
(Beni Suef 04)

Sheet (9)

Inscribed angles subtended by the same arc theorem (2), and its corollaries

Theorem (2):

In the same circle , the measures of all inscribed angles subtended by the same arc are equal.

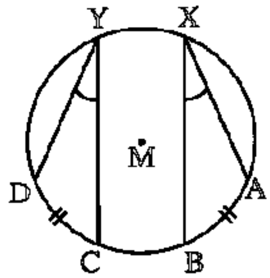


$\angle C$, $\angle D$ and $\angle E$ are inscribed angles subtended by $\overset{\frown}{AB}$

$m(\angle C) = m(\angle D) = m(\angle E)$

Corollary:

In the same circle (or in any number of circles) the measures of the inscribed angles subtended by arcs of equal measures are equal.

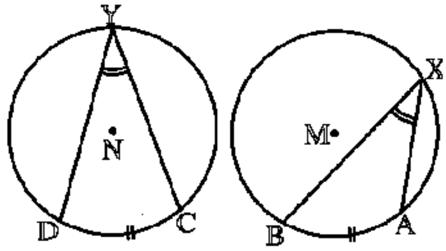


i.e. In the circle M

If $m(\overset{\frown}{AB}) = m(\overset{\frown}{CD})$,

then $m(\angle X) = m(\angle Y)$

∘ Notice that: In this case , the length of $\overline{AB}$ = the length of $\overline{CD}$

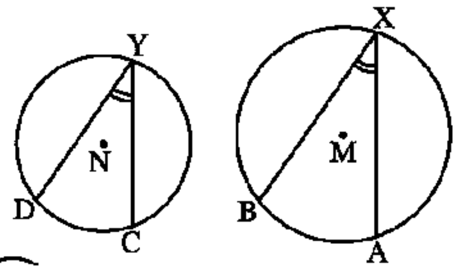


Also : If M and N are two congruent circles

and $m(\overset{\frown}{AB}) = m(\overset{\frown}{CD})$,

then $m(\angle X) = m(\angle Y)$

∘ Notice that: In this case , the length of $\overline{AB}$ = the length of $\overline{CD}$



Similarly : In any two circles M and N

If $m(\overset{\frown}{AB}) = m(\overset{\frown}{CD})$,

then $m(\angle X) = m(\angle Y)$

∘ Notice that: In this case , the length of $\overline{AB} \neq$ the length of $\overline{CD}$

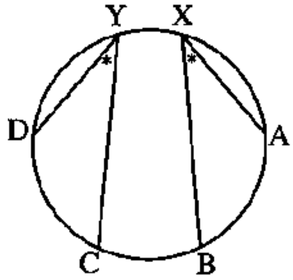
The converse of the previous Corollary is true also:

In the same circle (or in any number of circles) the inscribed angles of equal measures subtend arcs of equal measures.

In the opposite figure :

If $m(\angle X) = m(\angle Y)$,

then $m(\overset{\frown}{AB}) = m(\overset{\frown}{CD})$



Complete:

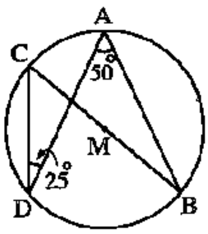
1 The inscribed angles subtended by the same arc in the same circle are ....

2 The inscribed angles subtended by equal arcs in measure in the same circle are ....

3 **In the opposite figure :**

$m(\angle C) =$ ....°

, $m(\angle B) =$ .... °

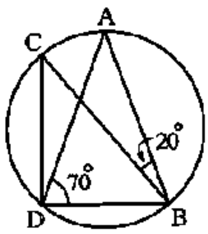


4 **In the opposite figure :**

If AB = AD , then

$m(\angle C) =$ ....°

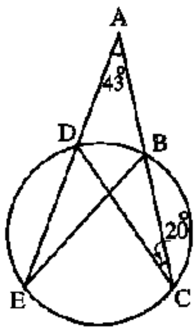
, $m(\angle BDC) =$ ....°



5 **In the opposite figure :**

$m(\angle BED) =$ .... °

, $m(\angle ABE) =$ .... °



Choose the correct answer :

1

In the opposite figure :

If m (∠ BAC) = 30° , then

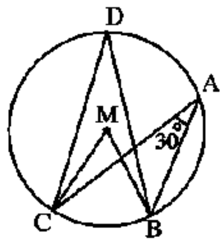
First : m (∠ BDC) = ...

(a) 15° (b) 30°

(c) 60° (d) 150°

Second : m (∠ BMC) = ...

(a) 30° (b) 90° (c) 60° (d) 120°



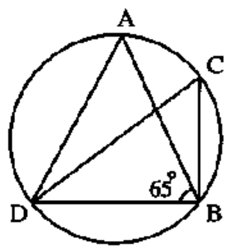
2

In the opposite figure :

If m (∠ ABD) = 65°

, AB = AD

, then m (∠ BCD) = ...



(Beni Suef 12)

(a) 15° (b) 25° (c) 30° (d) 50°

3

In the opposite figure :

A circle N , $\overline{XY}$ // $\overline{NZ}$

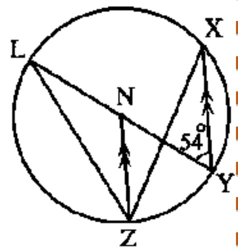
If m (∠ XYL) = 54° , then

First : m (∠ XZL) = ...

(a) 27° (b) 54° (c) 100° (d) 108°

Second : m (∠ YXZ) = ...

(a) 27° (b) 54° (c) 100° (d) 108°



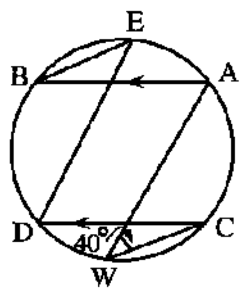
4

In the opposite figure :

$\overline{AB}$ // $\overline{CD}$, m (∠ AWC) = 40° ,

then m (∠ DEB) = ...

(a) 50° (b) 40° (c) 30° (d) 45°



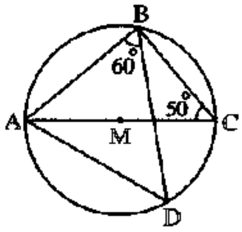
Essay problems:

1 **In the opposite figure :**

$\overline{AC}$ is a diameter in the circle M

, m (∠ C) = 50° , m (∠ ABD) = 60°

Find with proof : m (∠ CBD) and m (∠ BAD)

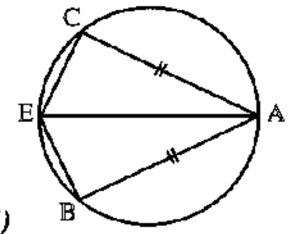


2 🕮 **In the opposite figure :**

AB = AC , E ∈ $\overset{\frown}{BC}$

Prove that :

m (∠ AEB) = m (∠ AEC) *(North Sinai 17 , Souhag 15)*

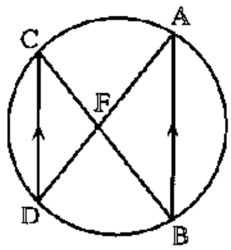


3 **In the opposite figure :**

$\overline{AB}$ and $\overline{CD}$ are two parallel chords in the circle

, $\overline{AD} \cap \overline{CB} = \{F\}$

Prove that : AF = FB

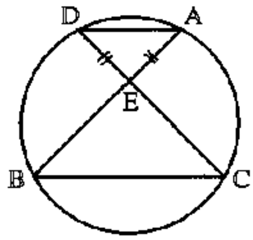


4 🕮 **In the opposite figure :**

$\overline{AB} \cap \overline{CD} = \{E\}$

, EA = ED

Prove that : EB = EC



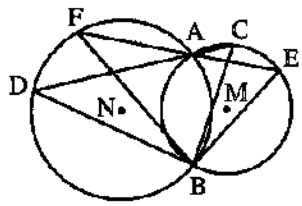
5 🕮 **In the opposite figure :**

M and N are two intersecting circles at A and B , $\overleftrightarrow{AC}$ intersects the circle M at C and intersects the circle N at D ,

$\overleftrightarrow{AE}$ intersects the circle M at E and intersects the circle N at F

Prove that : m (∠ EBC) = m (∠ FBD)

(Qena 17 , El-Beheira 13)



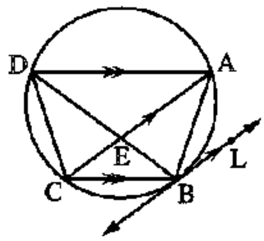
6 **In the opposite figure :**

ABCD is a quadrilateral inscribed in a circle where $\overline{BC}$ // $\overline{AD}$,

$\overline{AC} \cap \overline{BD} = \{E\}$

, $\overleftrightarrow{BL}$ is a tangent to the circle at B where $\overleftrightarrow{BL}$ // $\overline{AC}$

Prove that : (1) $\overrightarrow{DB}$ bisects ∠ ADC (2) m (∠ CBD) = m (∠ CDB)



Homework

Choose the correct answer :

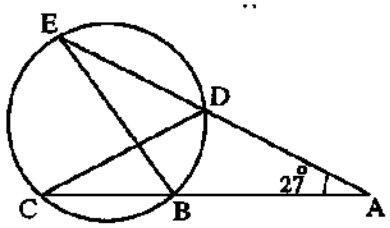
1 **In the opposite figure :**

$\overrightarrow{AD}$ intersects the circle at D and E ,

$\overrightarrow{AB}$ intersects it at B and C

If m (∠ A) = 27° , AB = BE , then m (∠ CDE) = ...

(a) 13.5° (b) 54° (c) 27° (d) 36°



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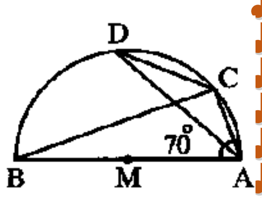
2 **In the opposite figure :**

A semicircle , m (∠ CAB) = 70°

, then m (∠ ADC) = ...

(a) 70° (b) 35° (c) 30° (d) 20°



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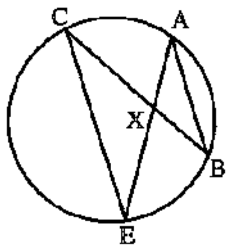
3 **In the opposite figure :**

The two triangles AXB and CXE are ....

(a) congruent. (b) similar.

(c) having the same perimeter. (d) having the same area.



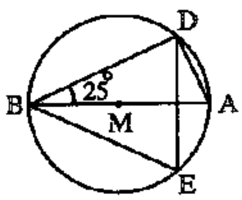
Essay problems:

1 **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, m (∠ ABD) = 25°

Find : m (∠ DEB) in degrees.



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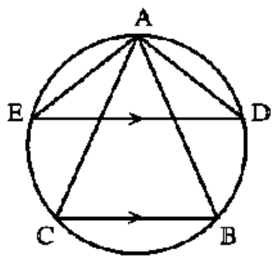
2 **In the opposite figure :**

ABC is a triangle inscribed in a circle ,

$\overline{DE}$ // $\overline{BC}$

Prove that : m (∠ DAC) = m (∠ BAE)

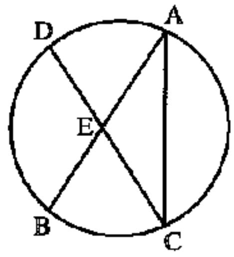


3 **In the opposite figure :**

$\overline{AB}$ and $\overline{CD}$ are two equal chords in length in the circle

, $\overline{AB} \cap \overline{CD} = \{E\}$

Prove that : The triangle ACE is an isosceles triangle.



(El-Kalyoubia 11)

4 **In the opposite figure :**

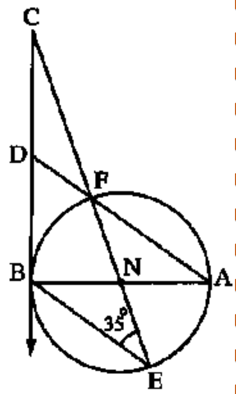
$\overline{AB}$ is a diameter in a circle of centre N ,

$\overrightarrow{CB}$ is a tangent to the circle at B ,

$\overrightarrow{CN}$ is drawn to cut the circle at F and E and $\overrightarrow{AF}$ is drawn to cut $\overrightarrow{CB}$ at D

If m (∠ BEC) = 35°

Find : (1) m (∠ BNC) (2) m (∠ BCN) (3) m (∠ BDA)

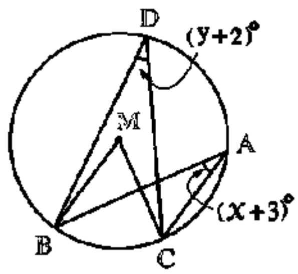


« 70° , 20° , 55° »

5 **In the opposite figure :**

M is a circle , ∠ A and ∠ D are two inscribed angles of measures $(x + 3)°$ and $(y + 2)°$ respectively. If $y^2 - x^2 = 53$

Find : m (∠ CMB)



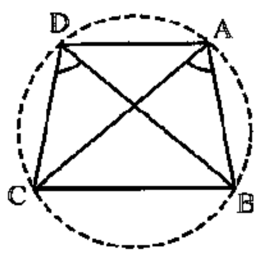
« 58° »

Sheet (10)
The cyclic quadrilateral
the converse of theorem (2)

The cyclic quadrilateral is a quadrilateral whose four vertices belong to one circle.

In the opposite figure :

If ABCD is a quadrilateral and we can draw a circle to pass through its four vertices A , B , C and D , then the figure ABCD is called a cyclic quadrilateral , then each two angles drawn on one of its sides as a base and their vertices are two vertices of the figure are equal in measure because they are inscribed angles subtended by the same arc.

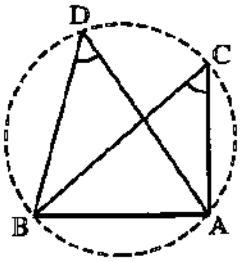


The converse of the theorem (2):

If two angles subtended by the same base and on the same side of it have the same measure , then their vertices are on an arc of a circle and the base is a chord of it.

In the opposite figure :

If ∠ C and ∠ D are drawn on the base $\overline{AB}$ and on the same side of it ,
m (∠ C) = m (∠ D) , then the points A , B , C and D lie on a unique circle ,
then $\overline{AB}$ is a chord of it.



Remarks

1. If there are two angles drawn on one of the sides of a quadrilateral , they are on the same side of it and they are not equal in measure , then the quadrilateral is not cyclic.
2. Each of the rectangle , the square and the isosceles trapezium are cyclic quadrilaterals while each of the parallelogram, the rhombus and the trapezium that is not isosceles are not cyclic quadrilaterals.

Essay problems:

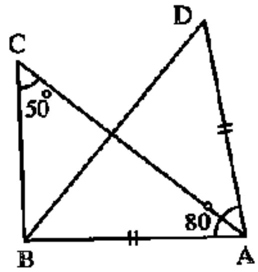
1 **In the opposite figure :**

AB = AD , m (∠ A) = 80°

and m (∠ C) = 50°

Prove that :

The points A , B , C and D have one circle passing through them.

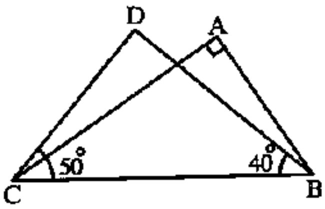


2 **In the opposite figure :**

m (∠ A) = 90° , m (∠ DBC) = 40° , m (∠ DCB) = 50°

(1) **Prove that :** The figure ABCD is a cyclic quadrilateral

(2) Determine where is the center of the circle passes through the vertices of the figure ABCD



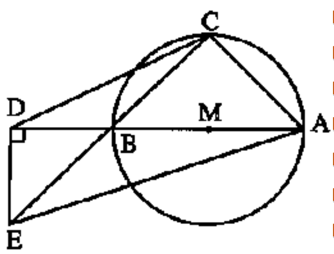
3 **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

Draw $\overleftrightarrow{DE} \perp \overleftrightarrow{AB}$ and $\overrightarrow{CB} \cap \overleftrightarrow{DE} = \{E\}$

Prove that :

ACDE is a cyclic quadrilateral.



(El-Fayoum 11)

4 **In the opposite figure :**

$\overline{AC}$ is a diameter in the circle M

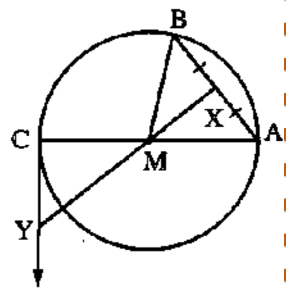
, X is the midpoint of $\overline{AB}$

and $\overrightarrow{CY}$ is a tangent to the circle cutting $\overrightarrow{XM}$ at Y

Prove that :

(1) The figure AXCY is a cyclic quadrilateral.

(2) m ($\angle$ BMC) = 2 m ($\angle$ MYC)



5 **In the opposite figure :**

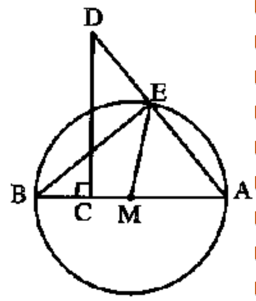
$\overline{AB}$ is a diameter in circle M in which

$\overleftrightarrow{AE}$ is a chord and $\overrightarrow{CD} \perp \overline{AB}$, $\overrightarrow{CD}$ intersects $\overrightarrow{AE}$ at D

Prove that :

(1) The points D , E , C and B have one circle passing through them.

(2) m ($\angle$ AME) = 2 m ($\angle$ D)



(Kafr El-Sheikh 11)

Homework

Choose the correct answer :

1 The inscribed angle which is subtended by major arc in a circle is ....

(a) reflex. (b) right. (c) obtuse. (d) acute.

2 If the length of an arc of a circle is $\frac{1}{3}$ π r cm. , then its opposite central angle of measure equals ....

(a) 30° (b) 60° (c) 120° (d) 240°

3 The ratio between the measure of the inscribed angle and the measure of the central angle that has the same subtended arc equals 2 : ....

(a) 1 (b) 3 (c) 4 (d) 6

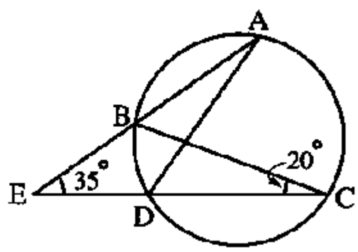
4 **In the opposite figure :**

If m ($\angle$ E) = 35° , m ($\angle$ C) = 20°

, then m ($\overset{\frown}{AC}$) = ...

(a) 135° (b) 110°

(c) 65° (d) 55°

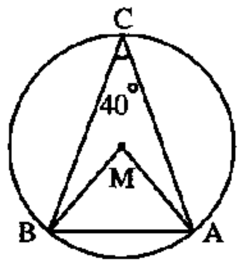


5 In the opposite figure :

m (∠ MAB) = ………

(a) 40° (b) 80°

(c) 70° (d) 50°



Essay problems:

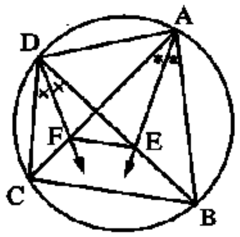
1 In the opposite figure :

ABCD is a cyclic quadrilateral which has $\overrightarrow{AE}$ bisects ∠ BAC and $\overrightarrow{DF}$ bisects ∠ BDC

Prove that :

(1) AEFD is a cyclic quadrilateral.

(2) $\overline{EF}$ // $\overline{BC}$



(Luxor 16 , El-Dakahlia 13)

2 ABCD is a square , $\overrightarrow{AX}$ bisects ∠ BAC and intersects $\overline{BD}$ at X , $\overrightarrow{DY}$ bisects ∠ CDB and intersects $\overline{AC}$ at Y

Prove that :

(1) AXYD is a cyclic quadrilateral.

(2) m (∠ AYX) = 45°

(Alexandria 16 , Sharkia 12)

3 In the opposite figure :

ABC is a triangle in which : AB = AC ,

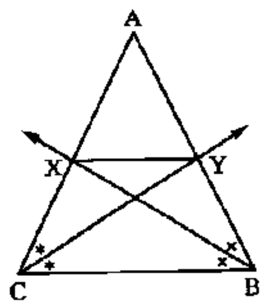
$\overrightarrow{BX}$ bisects ∠ B and intersects $\overline{AC}$ at X ,

$\overrightarrow{CY}$ bisects ∠ C and intersects $\overline{AB}$ at Y

Prove that :

(1) BCXY is a cyclic quadrilateral.

(2) $\overleftrightarrow{XY}$ // $\overleftrightarrow{BC}$

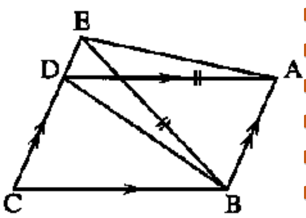


(El-Fayoum 17 , Assiut 11)

4 In the opposite figure :

ABCD is a parallelogram , E ∈ $\overrightarrow{CD}$, where BE = AD

Prove that : ABDE is a cyclic quadrilateral.



(Luxor 17 , S. Sinai 13)

Sheet (11)

Properties of the cyclic quadrilateral [theorem (3)]

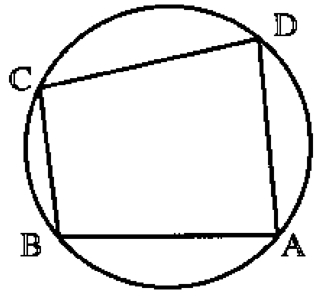
Theorem (3):

In a cyclic quadrilateral , each two opposite angles are supplementary.

ABCD is a cyclic quadrilateral

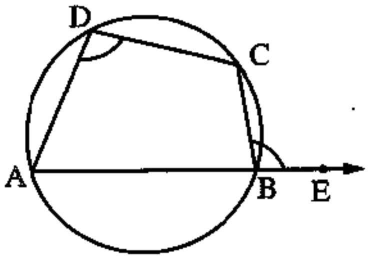
$m(\angle A) + m(\angle C) = 180°$

$m(\angle B) + m(\angle D) = 180°$



Corollary:

The measure of the exterior angle at a vertex of a cyclic quadrilateral is equal to the measure of the interior angle at the opposite vertex.



$m(\angle CBE) = m(\angle D)$

In each of the following figures , find the measure of the angle denoted by the sign (?) given that M is the centre of the circle :

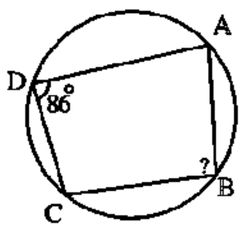


Fig. (1)

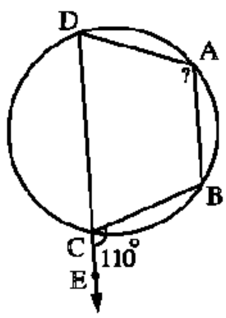


Fig. (2)

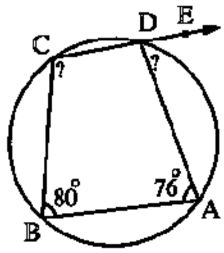


Fig. (3)

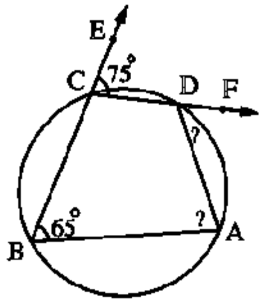


Fig. (4)

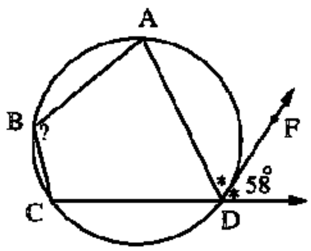


Fig. (5)

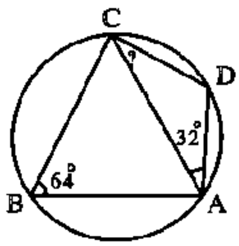


Fig. (6)

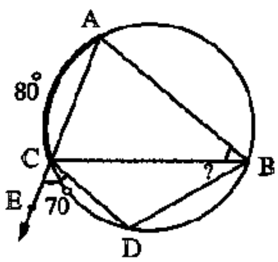


Fig. (7)

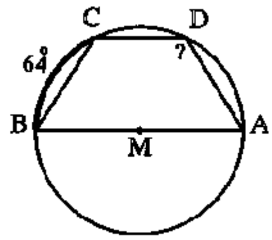


Fig. (8)

Complete:

1 If the quadrilateral is cyclic , then each two opposite angles in it are .... (Cairo 17)

2 The measure of the exterior angle at a vertex of the cyclic quadrilateral is equal to the measure of the .... angle. (Dakahlia 12)

3 In the cyclic quadrilateral ABCD , if m (∠ C) = 115° , then m (∠ A) = ...° (Alex. 05)

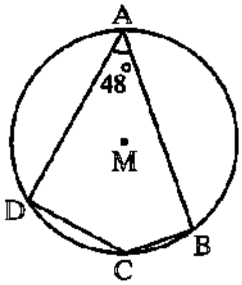
4 If the figure ABCD is a cyclic quadrilateral , m (∠ A) = 60° , then the measure of the exterior angle at the vertex C equals .... °

5 **In the opposite figure :**

If M is a circle , m (∠ A) = 48°

, then m (∠ C) = ... °

and m ($\overset{\frown}{BD}$ the major) = ... ° (New Valley 17)



6 If ABCD is a cyclic quadrilateral and m (∠ B) = $\frac{1}{4}$ m (∠ D) ,

then m (∠ B) = ... °

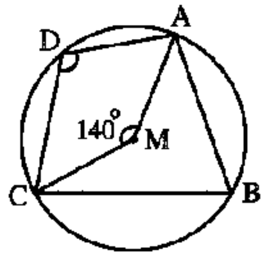
Choose the correct answer :

1 **In the opposite figure :**

In the circle M

, if m (∠ AMC) = 140°

, then m (∠ ADC) = ...



(a) 40° (b) 70° (c) 110° (d) 140°

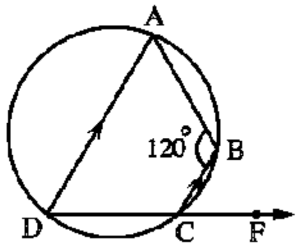
2 **In the opposite figure :**

If m (∠ B) = 120°

, $\overline{BC}$ // $\overline{AD}$

, then m (∠ BCF) = ...

(North Sinai 17)



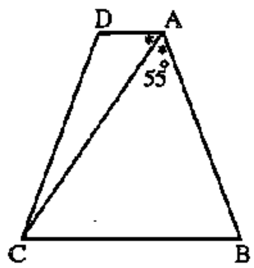
(a) 30° (b) 60° (c) 80° (d) 120°

3 In the opposite figure :

ABCD is a cyclic quadrilateral

in which $\overrightarrow{AC}$ bisects ∠ BAD ,

If m (∠ BAC) = 55°, then m (∠ BCD) = ………



(Cairo 05)

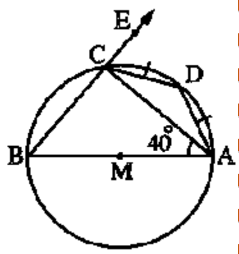
(a) 55° (b) 70° (c) 110° (d) 125°

4 In the opposite figure :

If $\overline{AB}$ is a diameter in the circle M ,

m (∠ BAC) = 40° , m ($\overset{\frown}{AD}$) = m ($\overset{\frown}{DC}$)

and E ∈ $\overrightarrow{BC}$, then :



First : m (∠ ACB) = ………

(a) 25° (b) 65° (c) 90° (d) 130°

Second : m (∠ ADC) = ………

(a) 25° (b) 65° (c) 90° (d) 130°

Third : m (∠ DAC) = ………

(a) 25° (b) 65° (c) 90° (d) 130°

Fourth : m (∠ DCE) = ………

(a) 25° (b) 65° (c) 90° (d) 130°

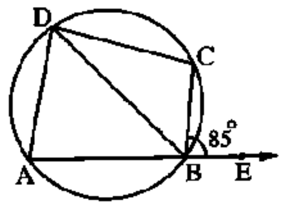
Essay problems:

1 **In the opposite figure :**

E ∈ $\overrightarrow{AB}$, E ∉ $\overline{AB}$, m ($\overset{\frown}{AB}$) = 110°

and m (∠ CBE) = 85°

Find : m (∠ BDC)

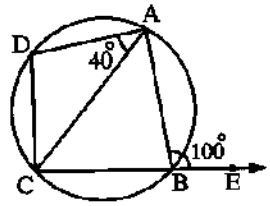


2 **In the opposite figure :**

m (∠ ABE) = 100°

and m (∠ CAD) = 40°

Prove that : m ($\overset{\frown}{CD}$) = m ($\overset{\frown}{AD}$)



3

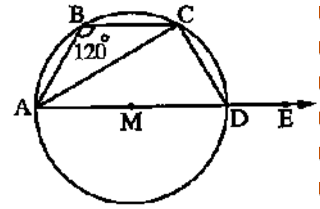
In the opposite figure :

ABCD is a quadrilateral inscribed in a circle M

where m (∠ B) = 120° , $\overline{AD}$ is a diameter in the circle , E ∈ $\overrightarrow{AD}$

(1) **Find :** m (∠ CDE) , m (∠ CAD)

(2) If DC = 7 cm. , **find :** The length of $\overset{\frown}{AD}$ $\left(\pi \approx \frac{22}{7}\right)$



« 120° , 30° , 22 cm. »

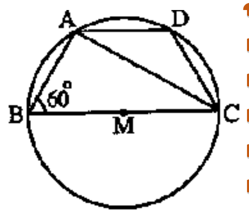
4

In the opposite figure :

ABCD is a cyclic quadrilateral , $\overline{CB}$ is a diameter in the circle M ,

m (∠ ABC) = 60° , the length of $\overset{\frown}{AD}$ = the length of $\overset{\frown}{CD}$

Prove that : $\overrightarrow{CA}$ bisects ∠ DCB



(Monofia 08)

5

In the opposite figure :

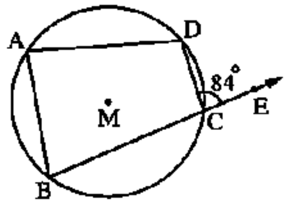
ABCD is a quadrilateral inscribed in the circle M

, E ∈ $\overrightarrow{BC}$, m (∠ DCE) = 84°

and m (∠ B) = $\frac{1}{2}$ m (∠ D)

Find :

(1) m (∠ A) (2) m (∠ B)



« 84° , 60° »

6

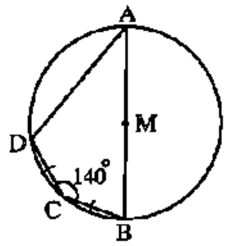
📖 **In the opposite figure :**

ABCD is a quadrilateral inscribed in a circle M where

M ∈ $\overline{AB}$, CB = CD and m (∠ BCD) = 140°

Find : (1) m (∠ A)

(2) m (∠ D)



(Matrouh 17 , Kafr El-Sheikh 14) « 40° , 110° »

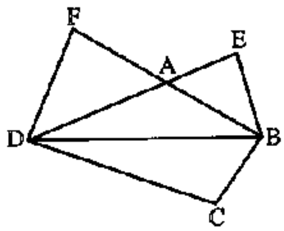
7

In the opposite figure :

EBCD is a cyclic quadrilateral

and FBCD is a cyclic quadrilateral

Prove that : The figure EBDF is a cyclic quadrilateral.



Homework

📖 **In each of the following figures , find the value of the symbol used in measure :**

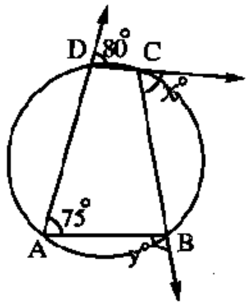


Fig. (1)

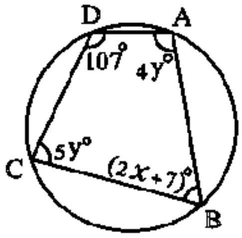


Fig. (2)

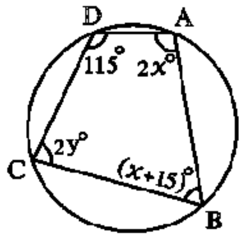


Fig. (3)

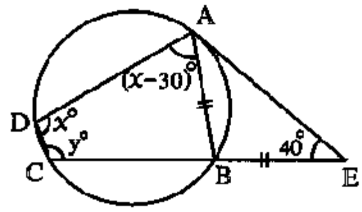


Fig. (4)

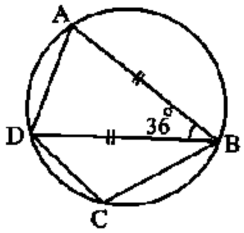
Choose the correct answer :

1 **In the opposite figure :**

If AB = BD

and m (∠ ABD) = 36°

, then m (∠ C) = ...



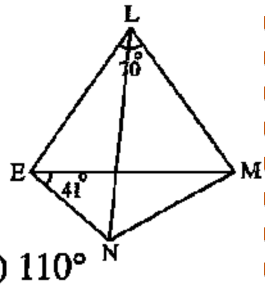
(a) 140° (b) 70° (c) 54° (d) 108°

2 **In the opposite figure :**

LMNE is a cyclic quadrilateral

, m (∠ MLE) = 70° , m (∠ MEN) = 41°

, then m (∠ EMN) = ...



(a) 70° (b) 41° (c) 29° (d) 110°

Essay problems:

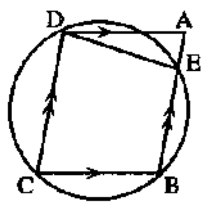
1 **In the opposite figure :**

ABCD is a parallelogram ,

the circle which passes through

the points B , C and D intersects $\overline{AB}$ at E

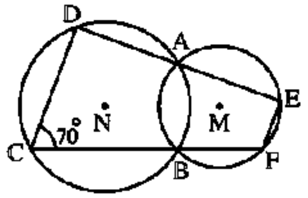
Prove that : AD = ED



(El-Fayoum 11)

2 **In the opposite figure :**

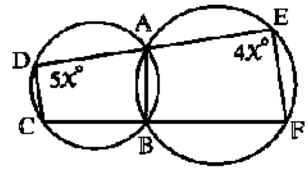
M and N are two intersecting circles at A and B ,
$\overleftrightarrow{AD}$ is drawn to intersect circle M at E and circle N at D ,
$\overleftrightarrow{BC}$ is drawn to intersect circle M at F and circle N at C
and m (∠ C) = 70°



(1) **Find :** m (∠ F) (2) **Prove that :** $\overleftrightarrow{CD}$ // $\overleftrightarrow{EF}$ *(El-Monofia 17)* « 110° »

3 **In the opposite figure :**

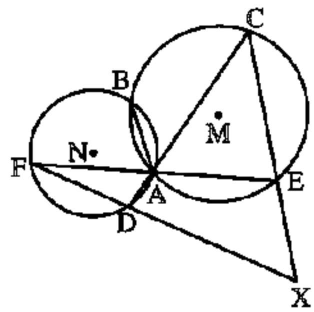
Two intersecting circles at A and B
, A ∈ $\overline{ED}$, B ∈ $\overline{FC}$, m (∠ D) = 5 x°
and m (∠ E) = 4 x°



Find with proof : m (∠ ABF) « 100° »

4 **In the opposite figure :**

$\overline{AB}$ is a common chord of the two circles M and N ,
C ∈ the circle M , F ∈ the circle N. If $\overrightarrow{CA}$ intersects
the circle N at D and $\overrightarrow{FA}$ intersects the circle M at E
, $\overrightarrow{CE}$ ∩ $\overrightarrow{FD}$ = {X} and the figure AEXD is a cyclic quadrilateral.
Prove that : C , B and F are collinear.



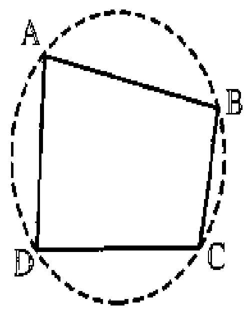
Sheet (12)
The converse of theroem (3) and its corollary

The converse of theorem (3):

If two opposite angles of a quadrilateral are supplementary , then the quadrilateral is cyclic.

In the opposite figure :

If m (∠ B) + m (∠ D) = 180° or m (∠ A) + m (∠ C) = 180°
, then the figure ABCD is a cyclic quadrilateral



Corollary:

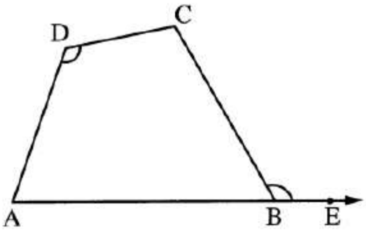
If the measure of the exterior angle at a vertex of a quadrilateral figure is equal to the measure of the interior angle at the opposite vertex, then the figure is a cyclic quadrilateral.

In the opposite figure :

If ABCD is a quadrilateral

and m (∠ CBE) (the exterior angle) = m (∠ D),

then the figure ABCD is a cyclic quadrilateral.



A summary of the cases in which the quadrilateral is cyclic :

The quadrilateral is cyclic if one of the following conditions is verified :

1. If there is a point in the plane of the figure such that it is equidistant from its vertices.
2. If there are two equal angles in measure and drawn on one of its sides as a base and on one side of this side.
3. If there are two opposite supplementary angles «their sum = 180°»
4. If there is an exterior angle at any of its vertices equal in measure to the measure of the interior angle at the opposite vertex.

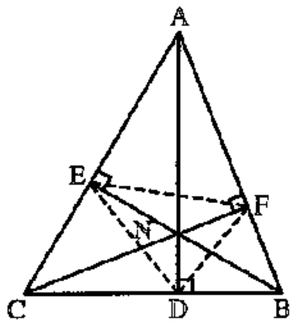
Remarks

In the opposite figure :

If $\overline{AD}$, $\overline{BE}$, $\overline{CF}$ are the altitudes of Δ ABC, then :

- $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ are concurrent at one point (say N)
- From the figure we can get six cyclic quadrilaterals, they are :

NFBD, NECD, NFAE, FBCE, DCAF and EABD



In each of the following figures, prove that the figure ABCD is a cyclic quadrilateral :

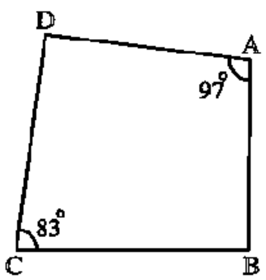


Fig. (1)

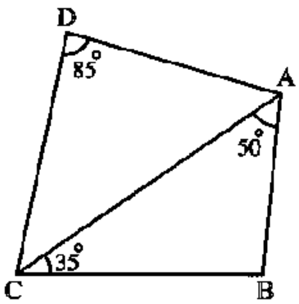


Fig. (2)

(South Sinai 16)

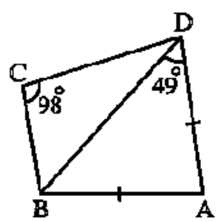


Fig. (3)

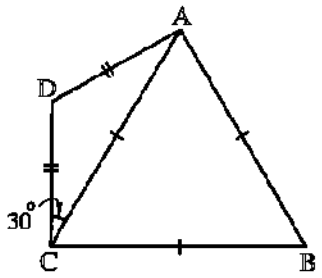


Fig. (4)
(Aswan 12)

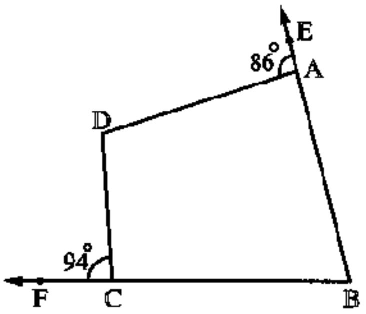


Fig. (5)

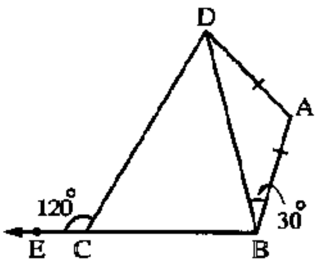


Fig. (6)
(Aswan 16 , Damietta 15)

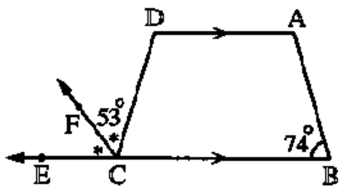


Fig. (7)
(Port Said 17 , Damietta 17)

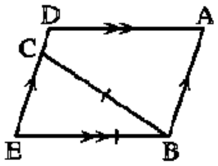


Fig. (8)

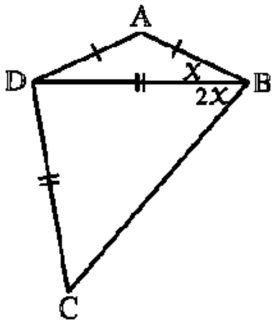


Fig. (9)
(El-Dakahlia 13)

Essay problems:

1 **In the opposite figure :**

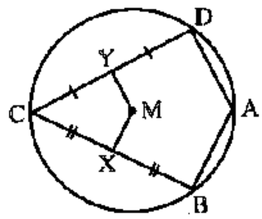
ABCD is a quadrilateral inscribed in a circle M

, X is the midpoint of $\overline{BC}$ and Y is the midpoint of $\overline{CD}$

Prove that :

(1) The figure MXCY is a cyclic quadrilateral.

(2) m (∠ XMY) = m (∠ BAD)



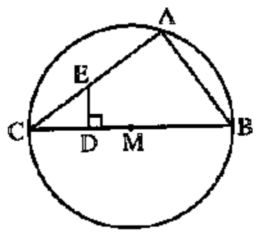
2 **In the opposite figure :**

$\overline{BC}$ is a diameter in the circle M and $\overline{ED} \perp \overline{BC}$

Prove that :

(1) The figure ABDE is a cyclic quadrilateral.

(2) m (∠ CED) = $\frac{1}{2}$ m $(\overset{\frown}{AC})$



(Giza 09)

3

In the opposite figure :

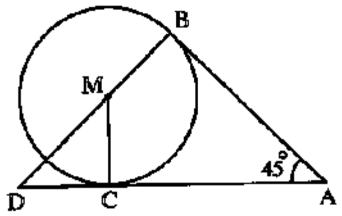
$\overline{AB}$ and $\overline{AC}$ touch the circle M at B and C respectively

, m ($\angle$ A) = 45°

Prove that :

(1) The figure ABMC is a cyclic quadrilateral.

(2) Δ MCD is an isosceles triangle.



(South Sinai 12)

4

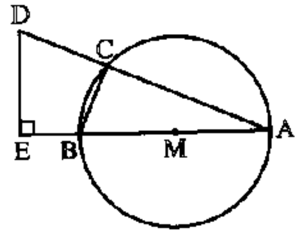
In the opposite figure :

$\overline{AB}$ is a diameter in the circle M

and D $\in \overrightarrow{AC}$. Draw $\overline{DE} \perp \overrightarrow{AB}$

Prove that :

The figure BEDC is a cyclic quadrilateral.



5

In the opposite figure :

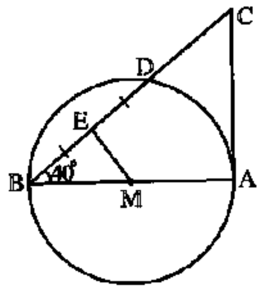
$\overline{AB}$ is a diameter in a circle of centre M

, $\overrightarrow{AC}$ is a tangent to the circle at A

, E is the midpoint of $\overline{DB}$, m ($\angle$ B) = 40°

(1) **Prove that :** The figure AMEC is a cyclic quadrilateral.

(2) **Find :** m ($\angle$ C)



(El-Wadi El-Gedied 14) « 50° »

6

In the opposite figure :

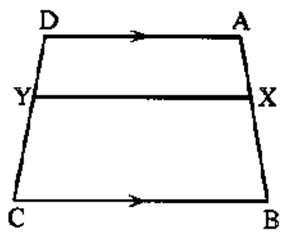
ABCD is a quadrilateral ,

$\overline{AD}$ // $\overline{BC}$, X $\in \overline{AB}$ and Y $\in \overline{DC}$

If the figure AXYD is a cyclic quadrilateral.

Prove that :

The figure XBCY is a cyclic quadrilateral.



Homework

Essay problems:

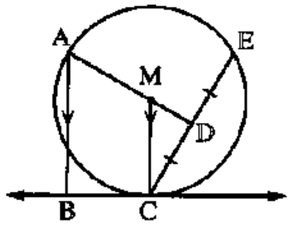
1 **In the opposite figure :**

M is a circle , D is the midpoint of the chord $\overline{EC}$

, $\overleftrightarrow{BC}$ is a tangent to the circle M at C and $\overline{AB}$ // $\overline{MC}$

Prove that :

The figure ABCD is a cyclic quadrilateral.



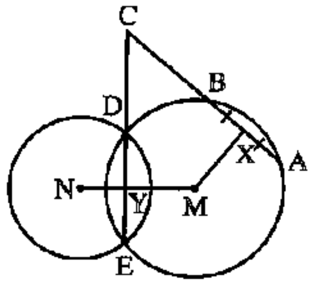
(Cairo 12)

2 **In the opposite figure :**

X is the midpoint of $\overline{AB}$, $\overline{MN} \cap \overline{EC} = \{Y\}$

(1) **Prove that :** CXMY is a cyclic quadrilateral

(2) Find the centre of the circle which passes through the vertices of the figure CXMY *(El-Ismailia 17)*



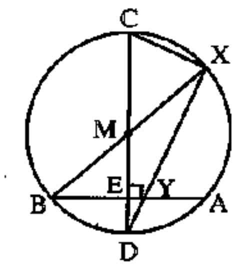
3 **In the opposite figure :**

$\overline{AB}$ is a chord in the circle M and $\overline{CD}$ is a perpendicular diameter on $\overline{AB}$ and intersects it at E

, $\overrightarrow{BM}$ intersects the circle at X and $\overline{XD} \cap \overline{AB} = \{Y\}$

Prove that : (1) XYEC is a cyclic quadrilateral.

(2) m (∠ DYB) = m (∠ DBX)

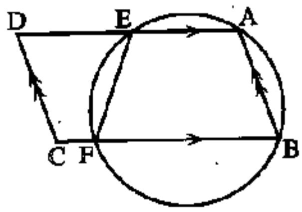


4 **In the opposite figure :**

ABCD is a parallelogram.

A circle is drawn to pass through the two points A and B to cut $\overline{AD}$ at E and $\overline{BC}$ at F

Prove that : The figure CDEF is a cyclic quadrilateral.



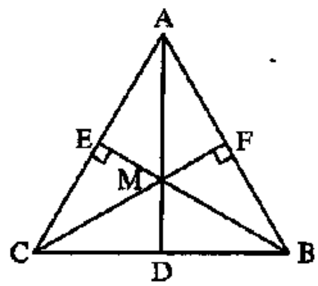
5 **In the opposite figure :**

Δ ABC , $\overline{BE} \perp \overline{AC}$, $\overline{CF} \perp \overline{AB}$, $\overline{CF} \cap \overline{BE} = \{M\}$

, $\overrightarrow{AM} \cap \overleftrightarrow{BC} = \{D\}$

Prove that :

MDCE is a cyclic quadrilateral. *(South Sinai 17)*



6 **In the opposite figure :**

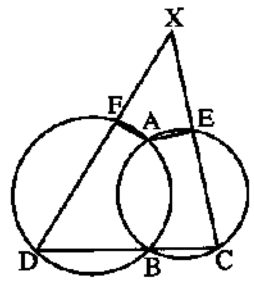
Two intersecting circles at A and B

, $\overline{CD}$ passes through the point B and intersects

the two circles at C and D

, $\overrightarrow{CE} \cap \overrightarrow{DF} = \{X\}$

Prove that : AFXE is a cyclic quadrilateral.



Sheet (13)

The relation between the tangents of a circle theorem (3) and its corollaries

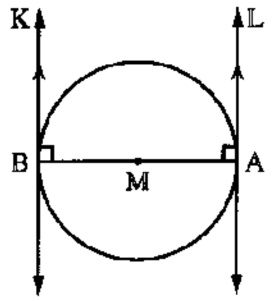
First : The two tangents drawn at the two ends of a diameter in a circle are parallel.

i.e. **In the opposite figure :**

If $\overline{AB}$ is a diameter in the circle M and the two straight lines L and K are two tangents to the circle at A and B respectively,

then the straight line L // the straight line K

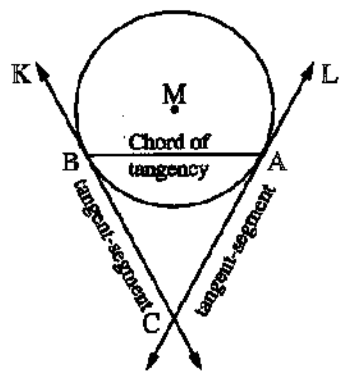
(because the straight line L $\perp$ $\overline{AB}$ and the straight line K $\perp$ $\overline{AB}$)



Second : The two tangents drawn at the two ends of a chord of a circle are intersecting.

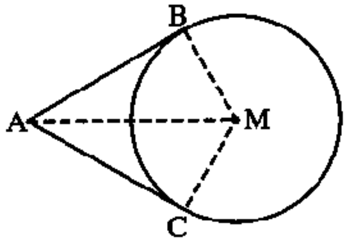
i.e. **In the opposite figure :**

If $\overline{AB}$ is a chord in the circle M and the two straight lines L and K are two tangents to the circle at A and B respectively, then the two straight lines L and K are intersecting at a point outside the circle M (Say C) and $\overline{AC}$, $\overline{BC}$ are called tangent-segments and $\overline{AB}$ is called a chord of tangency.



Theorem (4):

The two tangent-segments drawn to a circle from a point outside it are equal in length.



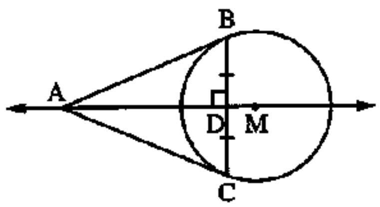
$\overline{AB}$ and $\overline{AC}$ are two tangent-segments

$AB = AC$

Corollary (1):

The straight line passing through the centre of the circle and the intersection point of the two tangents is an axis of symmetry to the chord of tangency of those two tangents.

In the opposite figure :



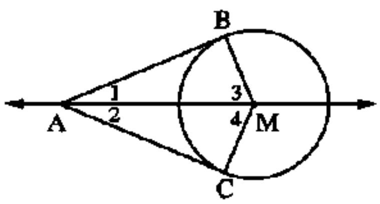
If $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ are two tangents to the circle M at B and C respectively, then $\overleftrightarrow{AM}$ is the axis of symmetry to $\overline{BC}$

i.e. $\overleftrightarrow{AM} \perp \overline{BC}$, $BD = CD$

Corollary (2):

The straight line passing through the centre of the circle and the intersection point of its two tangents bisects the angle between these two tangents. It also bisects the angle between the two radii passing through the two points of tangency.

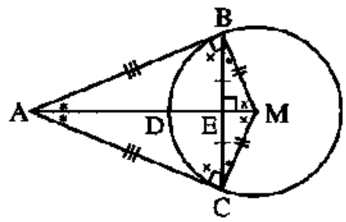
In the opposite figure :



If $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ are two tangents to the circle M at B and C respectively then :

- $\overrightarrow{AM}$ bisects $\angle BAC$ $\quad \therefore m(\angle 1) = m(\angle 2)$
- $\overrightarrow{MA}$ bisects $\angle BMC$ $\quad \therefore m(\angle 3) = m(\angle 4)$

Remarks:

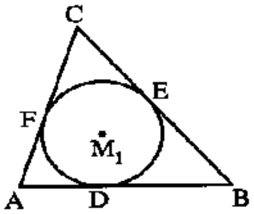


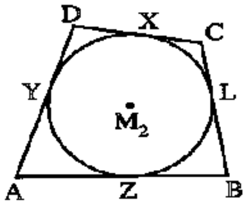
1. $AB = AC$
2. $MB = MC = r$
3. $BE = CE$, $\overleftrightarrow{AM} \perp \overline{BC}$
4. $m(\angle ABM) = m(\angle ACM) = 90°$

   i.e. The figure ABMC is a cyclic quadrilateral.
5. $m(\angle BAM) = m(\angle BCM) = m(\angle CAM) = m(\angle CBM)$
6. $m(\angle AMB) = m(\angle ACB) = m(\angle AMC) = m(\angle ABC)$

Definition:

The inscribed circle of a polygon is the circle which touches all of its sides internally.



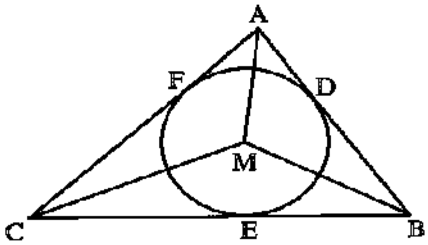


Remark:

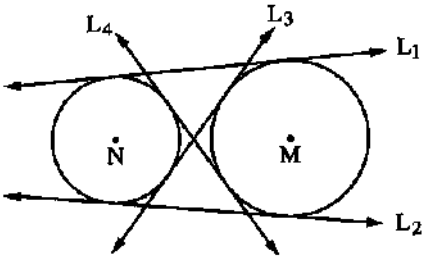
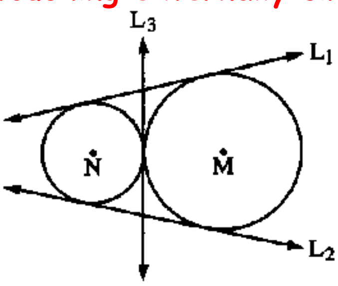
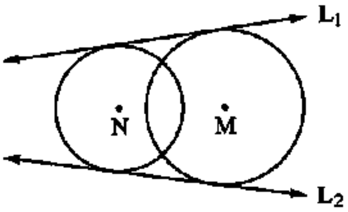
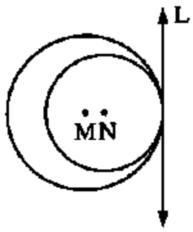
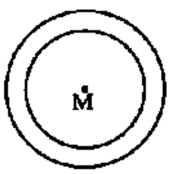
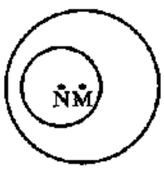
The centre of the inscribed circle of any triangle is the point of intersection of the bisectors of its interior angles.

In the opposite figure :

If the circle M is the inscribed circle of the triangle ABC then M is the intersection point of the bisectors of the interior angles of Δ ABC



The common tangents to two circles:

Two distant circles	Two touching externally circles
 **4 common tangents**	 **3 common tangents**
Two intersecting circles	**Two touching internally circles**
 **2 common tangents**	 **One common tangent**
One circle inside the other	**Concentric circles**
 **There are no common tangents**	 **There are no common tangents**

Complete:

1 The two tangents drawn to the circle at the two ends of a diameter in it are ....

2 The two tangent-segments drawn to a circle from a point outside it are .... *(Alex. 11)*

3 The inscribed circle of a triangle is ....

4 If $\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M at B and C , then $\overleftrightarrow{MA}$ is the axis of symmetry of ....

5 The number of common tangents of two distant circles is .... *(New Valley 12)*

6 The number of internal common tangents of the two intersecting circles is ....

7 The straight line passing through the centre of a circle and the point of intersection of two tangents to it is the axis of symmetry of ....

Choose the correct answer :

1 The number of tangents can be drawn from a point lies on a circle is ....

(El-Beheira 17)

(a) one (b) two (c) four (d) infinite number

2 $\overline{AB}$ and $\overline{AC}$ are two tangent-segments at the two points B and C to a circle of radius length 2 cm. If the length of $\overline{AB}$ = 5 cm. , then the length of $\overline{AC}$ = ... cm.

(a) 2 (b) 3 (c) 5 (d) 8

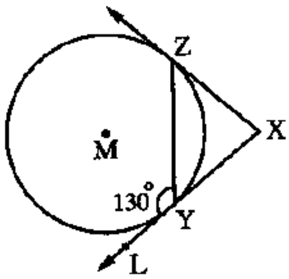
3 **In the opposite figure :**

$\overrightarrow{XY}$ and $\overrightarrow{XZ}$ are two tangents to the circle at Y and Z

, m ($\angle$ LYZ) = 130° , then m ($\angle$ X) = ...

(a) 50° (b) 65°

(c) 80° (d) 100°



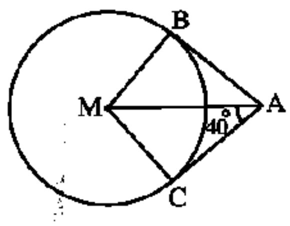
4 **In the opposite figure :**

If $\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M

, m ($\angle$ MAC) = 40° , then m ($\angle$ CAB) = ...

(a) 80° (b) 50°

(c) 40° (d) 20°



(Damietta 04)

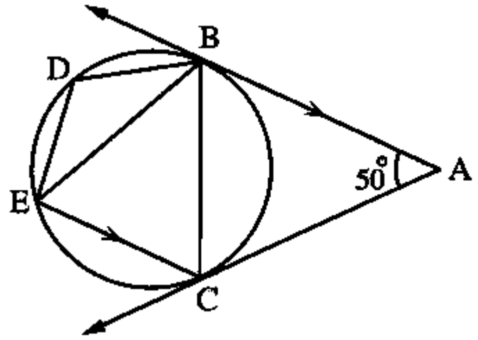
Essay problems:

1 **In the opposite figure :**

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ touch the circle at B and C ,

$\overrightarrow{AB}$ // $\overline{CE}$, m (∠ A) = 50°

Find by proof : m (∠ BDE)

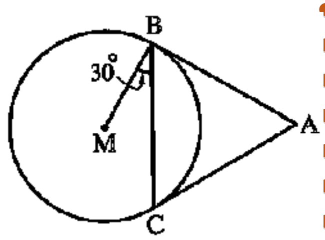


2 **In the opposite figure :**

If $\overline{AB}$ and $\overline{AC}$ are two tangent - segments to the circle M

and m (∠ MBC) = 30°

Prove that : Δ ABC is equilateral.

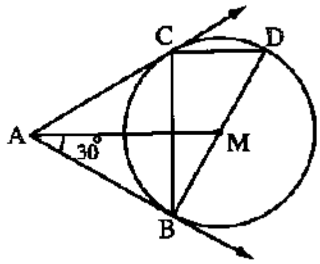


3 **In the opposite figure :**

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle M

, $\overline{BD}$ is a diameter in it , m (∠ MAB) = 30°

Find : m (∠ ACD)



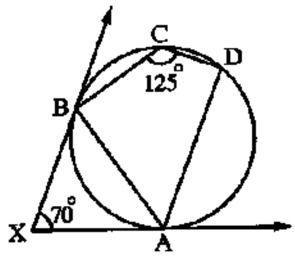
4 📖 **In the opposite figure :**

$\overrightarrow{XA}$ and $\overrightarrow{XB}$ are two tangents to the circle at A and B

, m (∠ AXB) = 70° and m (∠ DCB) = 125°

Prove that :

(1) $\overrightarrow{AB}$ bisects ∠ DAX (2) $\overline{AD}$ // $\overrightarrow{XB}$



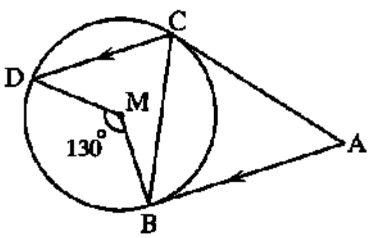
5 📖 **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M

, $\overline{AB}$ // $\overleftrightarrow{CD}$ and m (∠ BMD) = 130°

(1) **Prove that :** $\overrightarrow{CB}$ bisects ∠ ACD

(2) **Find :** m (∠ A) *(El-Fayoum 17 , El-Gharbia 16 , El-Kalyoubia 16 , El-Menia 15 , Cairo 14)* « 50° »



Homework

Choose the correct answer :

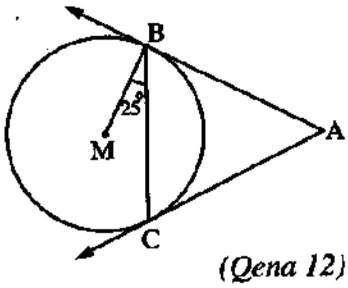
1 **In the opposite figure :**

If $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle M

, m (∠ CBM) = 25° , then m (∠ BAC) = ...

(a) 75° (b) 50°

(c) 25° (d) 12° 30̃



(Qena 12)

2 **In the opposite figure :**

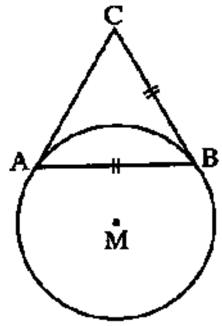
$\overline{CB}$ and $\overline{CA}$ are two tangent - segments

to the circle M and CB = BA

, then m (∠ C) = ...

(a) 60° (b) 120°

(c) 90° (d) 100°



(Suez 08)

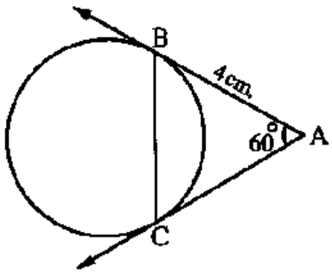
3 **In the opposite figure :**

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents , if AB = 4 cm.

, m (∠ A) = 60° , then BC = ...

(a) 3 cm. (b) 4 cm.

(c) 5 cm. (d) 8 cm.



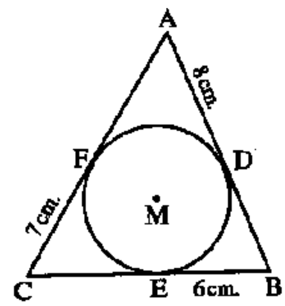
4 **In the opposite figure :**

The circle M touches the sides of Δ ABC , if AD = 8 cm. ,

BE = 6 cm. and CF = 7 cm. , then the perimeter of Δ ABC = ...

(a) 21 cm. (b) 42 cm.

(c) 48 cm. (d) 28 cm.



Essay problems:

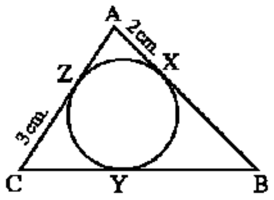
1 **In the opposite figure :**

Δ ABC touches the circle externally at X , Y and Z

If the perimeter of Δ ABC = 18 cm.

, AX = 2 m. and CZ = 3 cm.

Calculate : The length of $\overline{BY}$



(Sharkia 03) « 4 cm. »

2 **In the opposite figure :**

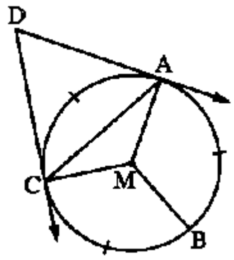
The circle M is divided into three arcs equal in length

, $\overrightarrow{DA}$ and $\overrightarrow{DC}$ are drawn from the point D to touch the circle.

(1) **Find :** m ($\angle$ AMB) « 120° »

(2) **Prove that : First :** The figure AMCD is a cyclic quadrilateral.

Second : Δ ACD is an equilateral triangle.



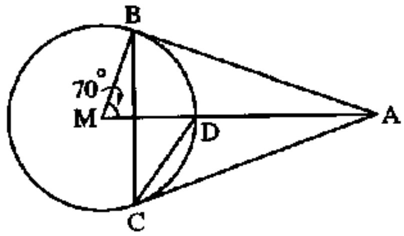
3 **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments drawn from A

, m ($\angle$ AMB) = 70°

Find : (1) m ($\angle$ ABC)

(2) m ($\angle$ ACD)



4 **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to

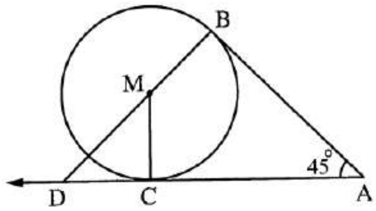
the circle M at B and C respectively , m ($\angle$ A) = 45°

, $\overrightarrow{BM} \cap \overrightarrow{AC} = \{D\}$

Prove that :

(1) The figure ABMC is cyclic quadrilateral.

(2) AD = AB + MB



(Helwan 09)

5 **In the opposite figure :**

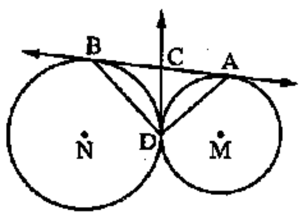
M and N are two circles touching externally at D and $\overleftrightarrow{AB}$ is a common tangent to them at A and B

$\overrightarrow{DC}$ is a common tangent to the two circles at D ,

where $\overrightarrow{DC} \cap \overleftrightarrow{AB} = \{C\}$

Prove that : (1) C is the midpoint of $\overline{AB}$

(2) $\overline{AD} \perp \overline{BD}$



(Alex. 14 , South Sinai 12)

Sheet (14)

Angles of tangency

theorem (5), its corollaries, and its converse

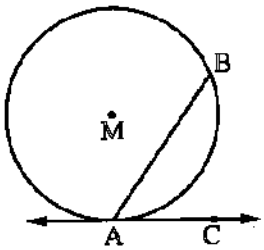
Definition:

The angle of tangency is the angle which is composed of the union of two rays , one of them is a tangent to the circle and the other contains a chord of the circle passing through the point of tangency.

In the opposite figure :

If $\overleftrightarrow{AC}$ is a tangent to the circle at A and $\overrightarrow{AB}$ contains the chord $\overline{AB}$, then $\angle$ BAC is an angle of tangency in the circle M , its chord is $\overline{AB}$

$\overline{AB}$ is called the chord of tangency of the angle of tangency $\angle$ BAC



The measure of the tangent angle:

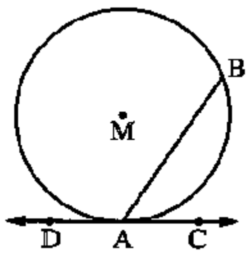
In the opposite figure :

- $\angle$ BAC is an angle of tangency that intercepts $\overset{\frown}{AB}$ between its sides.

  $\therefore m(\angle BAC) = \frac{1}{2} m(\overset{\frown}{AB})$

- $\angle$ BAD is an angle of tangency that intercepts the major $\overset{\frown}{AB}$ between its sides.

  $\therefore m(\angle BAD) = \frac{1}{2} m(\overset{\frown}{AB} \text{ the major})$

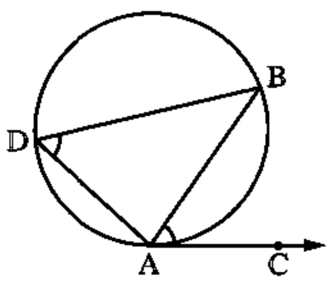


Theorem (5):

The measure of the angle of tangency is equal to the measure of the inscribed angle subtended by the same arc.

$\angle$ BAC is an angle of tangency and $\angle$ D is an inscribed angle.

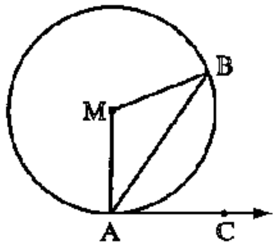
$m(\angle BAC) = m(\angle D)$



Corollary:

The measure of the angle of tangency is half the measure of the central angle subtended by the same arc.

In the opposite figure :



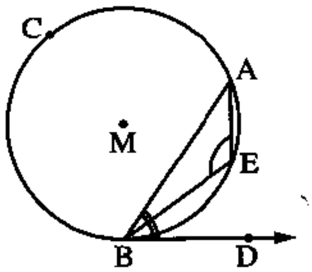
$m(\angle BAC)$ (tangency angle) $= \frac{1}{2} m(\widehat{AB})$

$, \because m(\angle AMB)$ (central angle) $= m(\widehat{AB})$

$\therefore m(\angle BAC)$ (tangency angle) $= \frac{1}{2} m(\angle AMB)$ (central angle)

Remark:

The angle of tangency is supplementary to the drawn inscribed angle on the chord of the angle of tangency and in one side of it.

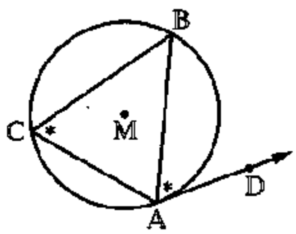


$m(\angle ABD) + m(\angle AEB) = 180°$

The converse of the theorem (5):

If a ray is drawn from one end of a chord of a circle so that the angle between this ray and the chord is equal in measure to the inscribed angle subtended by the chord in the alternate side , then this ray is a tangent to the circle.

Thus in the opposite figure :



If $\overline{AB}$ is a chord in the circle M ,

$\overrightarrow{AD}$ is drawn such that $m(\angle BAD) = m(\angle C)$,

then $\overrightarrow{AD}$ is a tangent to the circle M

In each of the following , find the measures of the angles denoted by (?) knowing that $\overrightarrow{AC}$ touches the circle M at A :

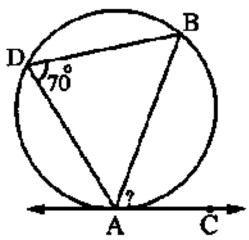


Fig. (1)

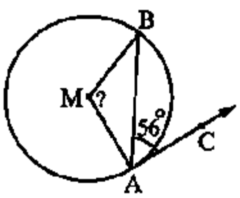


Fig. (2)

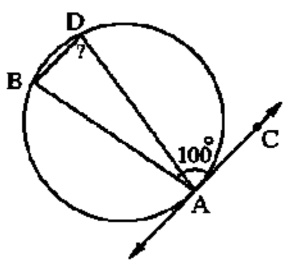


Fig. (3)

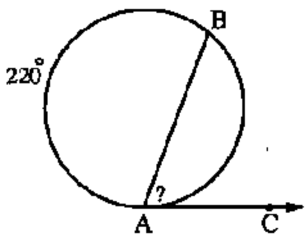


Fig. (4)

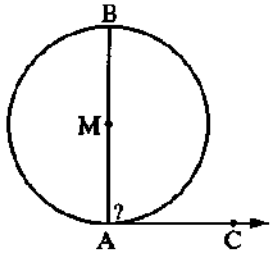


Fig. (5)

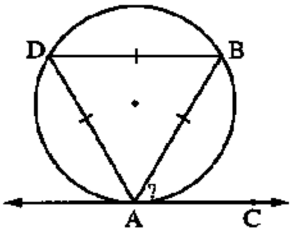


Fig. (6)

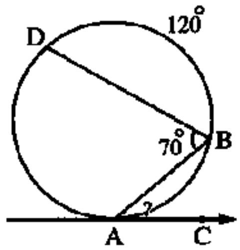


Fig. (7)

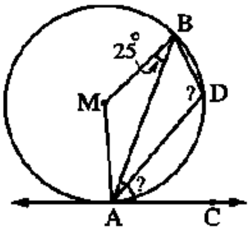


Fig. (8)

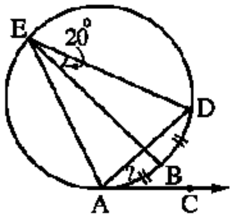


Fig. (9)

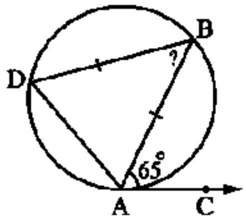


Fig. (10)

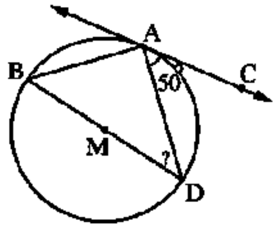


Fig. (11)

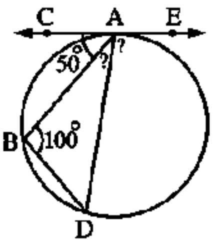


Fig. (12)

Choose the correct answer :

1 If the measure of an angle of tangency = 70° , then the measure of the central angle subtended by the same arc equals .... *(Aswan 2013)*

(a) 35°   (b) 70°   (c) 140°   (d) 105°

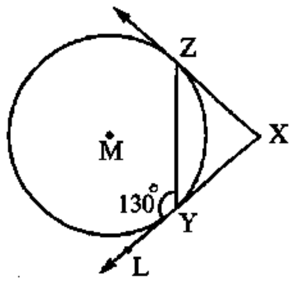
2 **In the opposite figure :**

$\overrightarrow{XY}$ and $\overrightarrow{XZ}$ are two tangents to the circle at Y and Z

, m (∠ LYZ) = 130° , then m (∠ X) = ...

(a) 50°   (b) 65°

(c) 80°   (d) 100°



(Souhag 2009)

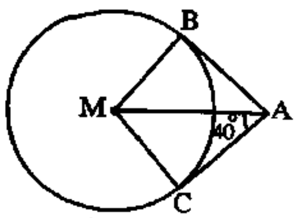
3 **In the opposite figure :**

If $\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M

, m (∠ MAC) = 40° , then m (∠ CAB) = ...

(a) 80°   (b) 50°

(c) 40°   (d) 20°



(Damietta 2004)

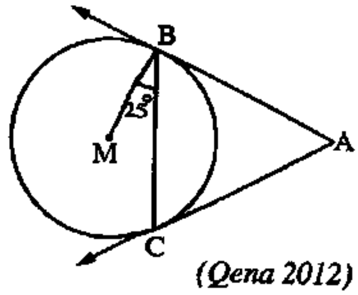
4 **In the opposite figure :**

If $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle M

, m (∠ CBM) = 25° , then m (∠ BAC) = ...

(a) 75° (b) 50°

(c) 25° (d) 12° 30́



(Qena 2012)

5 **In the opposite figure :**

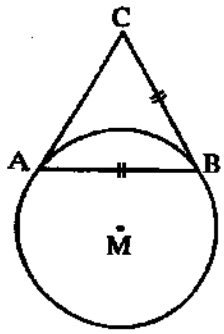
$\overline{CB}$ and $\overline{CA}$ are two tangent - segments

to the circle M and CB = BA

, then m (∠ C) = ...

(a) 60° (b) 120°

(c) 90° (d) 100°



(Suez 2008)

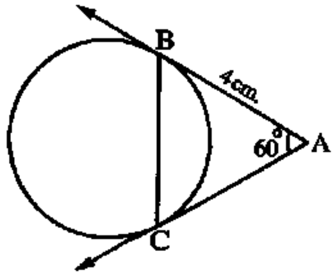
6 **In the opposite figure :**

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents , if AB = 4 cm.

, m (∠ A) = 60° , then BC = ...

(a) 3 cm. (b) 4 cm.

(c) 5 cm. (d) 8 cm.



(Port Said 2013)

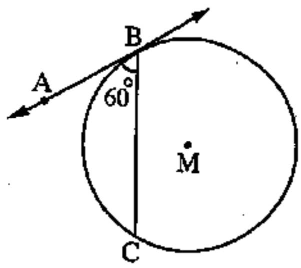
7 **In the opposite figure :**

$\overleftrightarrow{AB}$ is a tangent to the circle M at B

, $\overline{BC}$ is a chord in the circle

and m (∠ ABC) = 60°

, then m $(\overparen{BC})$ = ...



(Kafr El-Sheikh 2008)

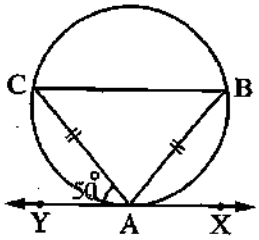
(a) 30° (b) 60° (c) 90° (d) 120°

8 In the opposite figure :

If AB = AC

and m (∠ YAC) = 50°

, then m $(\overset{\frown}{BC})$ = ...



(Cairo 2004)

50° (b) 100° (c) 80° (d) 160°

Essay problems:

1 In the opposite figure :

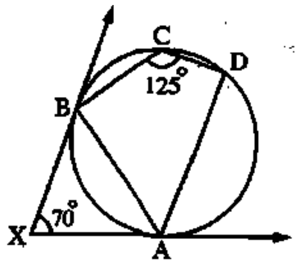
$\overrightarrow{XA}$ and $\overrightarrow{XB}$ are two tangents to the circle at A and B

, m (∠ AXB) = 70° and m (∠ DCB) = 125°

Prove that :

1 $\overrightarrow{AB}$ bisects ∠ DAX

2 $\overline{AD}$ // $\overrightarrow{XB}$



(El-Beheira 2011)

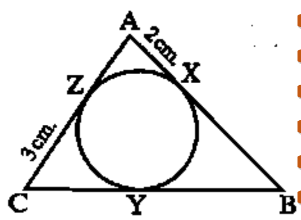
2 In the opposite figure :

Δ ABC touches the circle externally at X , Y and Z

If the perimeter of Δ ABC = 18 cm.

, AX = 2 m. and CZ = 3 cm.

Calculate : The length of $\overline{BY}$



(Sharkia 2003) « 4 cm. »

3 In the opposite figure :

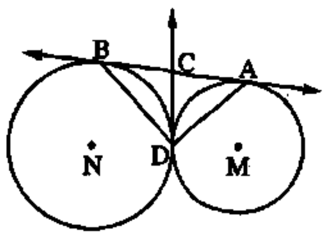
M and N are two circles touching externally at D and $\overleftrightarrow{AB}$ is a common tangent to them at A and B

$\overrightarrow{DC}$ is a common tangent to the two circles at D ,

where $\overrightarrow{DC} \cap \overleftrightarrow{AB} = \{C\}$

Prove that : 1 C is the midpoint of $\overline{AB}$

2 $\overline{AD} \perp \overline{BD}$



(Alex. 2014 , South Sinai 2012)

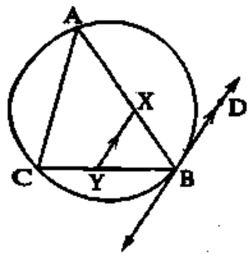
4 **In the opposite figure :**

ABC is a triangle inscribed in a circle

, $\overleftrightarrow{BD}$ is a tangent to the circle at B

, $X \in \overline{AB}$ and $Y \in \overline{BC}$, where $\overline{XY} \parallel \overleftrightarrow{BD}$

Prove that : AXYC is a cyclic quadrilateral.



(El-Kalyoubia 2014 , Port Said 2013)

5 **In the opposite figure :**

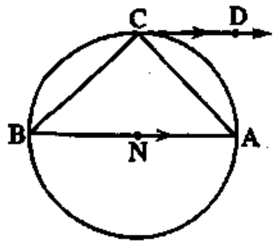
$\overline{AB}$ is a diameter in a circle N

, its circumference is 44 cm.

, $\overrightarrow{CD}$ is a tangent to it at C and $\overrightarrow{CD} \parallel \overline{BA}$

Find with proof :

1 m (∠ DCA) **2** The length of $\overset{\frown}{AC}$



(El-Fayoum 2013) « 45° , 11 cm. »

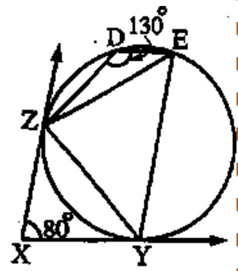
6 **In the opposite figure :**

$\overrightarrow{XY}$, $\overrightarrow{XZ}$ are two tangents to the circle at Y and Z

, m (∠ YXZ) = 80°

and m (∠ EDZ) = 130°

Prove that : **1** ZE = ZY **2** $\overline{XZ} \parallel \overline{YE}$



(Giza 2009)

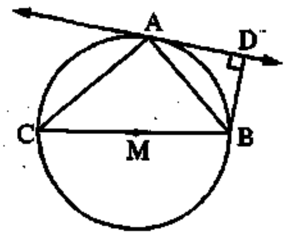
7 **In the opposite figure :**

$\overleftrightarrow{AD}$ is a tangent to the circle M at A

, $\overline{BC}$ is a diameter in the circle M

and $\overline{BD} \perp \overleftrightarrow{AD}$

Prove that : m (∠ ABD) = m (∠ ABC)



(Port Said 2006)

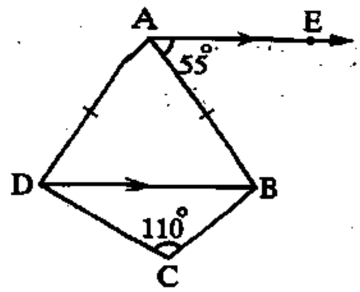
8 **In the opposite figure :**

$\overrightarrow{AE} \parallel \overline{DB}$, m (∠ BAE) = 55° ,

m (∠ C) = 110° and AB = AD

Prove that : **1** The figure ABCD is a cyclic quadrilateral.

2 $\overrightarrow{AE}$ is a tangent to the circumcircle of the quadrilateral ABCD



(Beheira 2005)

9 ABCD is a quadrilateral inscribed in a circle , E is a point outside the circle and $\overrightarrow{EA}$ and $\overrightarrow{EB}$ are two tangents to the circle at A and B , If m (∠ AEB) = 70° and m (∠ ADC) = 125° , **Prove that :**

1 AB = AC

2 $\overleftrightarrow{AC}$ is a tangent to the circle passing through the points A , B and E *(Alexandria 2014)*

10 ABCD is a parallelogram in which AC = BC

Prove that : $\overleftrightarrow{CD}$ is a tangent to the circle circumscribed about the triangle ABC

(Suez 2012 , Red Sea 2012 , El-Menia 2013)

Homework

Choose the correct answer :

1 If the measure of an angle of tangency = 70° , then the measure of the central angle subtended by the same arc equals .... *(El-Kalyoubia 16 , Aswan 13)*

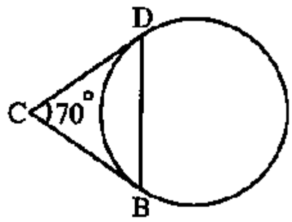
(a) 35° (b) 70° (c) 140° (d) 105°

2 **In the opposite figure :**

$\overline{CB}$, $\overline{CD}$ are two tangent-segments at B , D

, m (∠ C) = 70°

, then m ($\overset{\frown}{DB}$) equals ....



(El-Dakahlia 17)

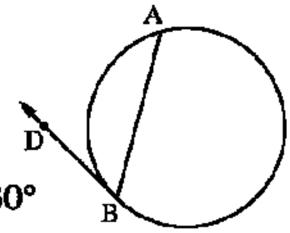
(a) 180° (b) 90° (c) 100° (d) 110°

3 **In the opposite figure :**

$\overrightarrow{BD}$ touches the circle and m ($\overset{\frown}{AB}$) = $\frac{1}{3}$ the measure of the circle

, then m (∠ ABD) = ...

(a) 60° (b) 90° (c) 120° (d) 30°

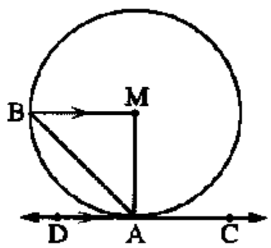


4 **In the opposite figure :**

$\overleftrightarrow{CD}$ is a tangent to the circle M at A

and $\overline{MB}$ // $\overleftrightarrow{CD}$, then m (∠ BAD) = ...

(a) 30° (b) 45° (c) 60° (d) 90°



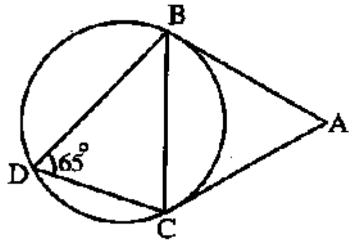
Essay problems:

1 **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle at B and C

, m (∠ BDC) = 65°

Find with proof : m (∠ BAC)



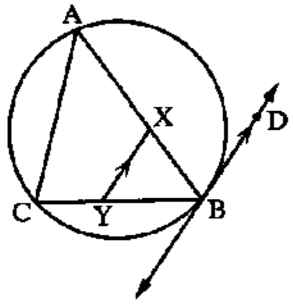
2 📖 **In the opposite figure :**

ABC is a triangle inscribed in a circle

, $\overleftrightarrow{BD}$ is a tangent to the circle at B

, X ∈ $\overline{AB}$ and Y ∈ $\overline{BC}$, where $\overline{XY}$ // $\overleftrightarrow{BD}$

Prove that : AXYC is a cyclic quadrilateral.



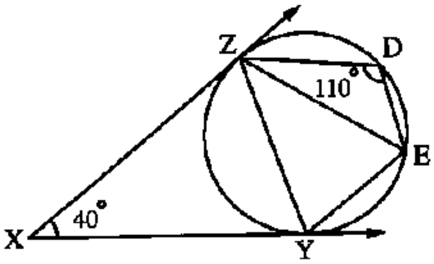
(Cairo 17 , El-Kalyoubia 14 , Port Said 13)

3 **In the opposite figure :**

$\overrightarrow{XY}$ and $\overrightarrow{XZ}$ are two tangents to the circle from the point X

, m (∠ D) = 110° , m (∠ X) = 40°

Prove that : m ($\overset{\frown}{ZDE}$) = m ($\overset{\frown}{ZY}$)



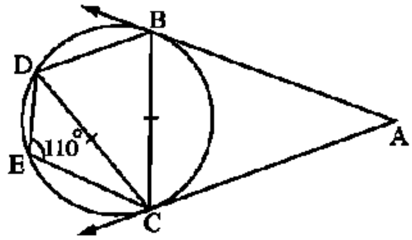
4 **In the opposite figure :**

$\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ are two tangents to the circle at B and C

If CB = CD

Prove that : m (∠ ABC) = m (∠ DBC)

If m (∠ CED) = 110° **Find :** m (∠ A)



« 40° »

5 **In the opposite figure :**

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ touch the circle at B and C

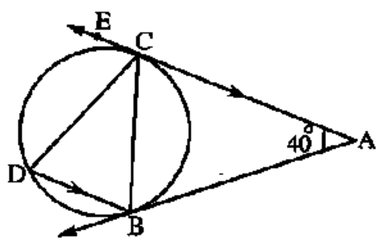
, $\overline{AC}$ // $\overline{BD}$ and m (∠ A) = 40°

Find with proof :

(1) m (∠ ACB)

(2) m (∠ ECD)

Then prove that : CB = CD



(Gharbia 04) « 70° , 70° »

6 **In the opposite figure :**

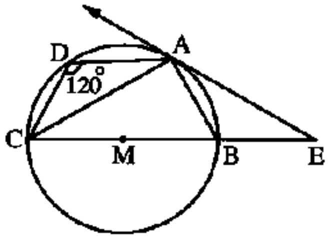
ABCD is a cyclic quadrilateral ,

$\overline{BC}$ is a diameter ,

$\overrightarrow{EA}$ is a tangent for the circle at point A

and m ($\angle$ ADC) = 120°

Prove that : (1) BA = BE (2) m ($\angle$ ABE) = m ($\angle$ EAC) *(Damietta 09)*



7 **In the opposite figure :**

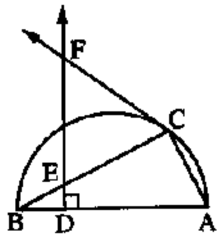
$\overline{AB}$ is a diameter of the semicircle ,

$\overrightarrow{CF}$ is a tangent to it at C and $\overrightarrow{DF} \perp \overline{AB}$

(1) **Prove that :** The figure ADEC is a cyclic quadrilateral.

(2) **Prove that :** Δ FCE is isosceles.

(3) Determine the centre of the circle passing through the vertices of the quadrilateral ADEC *(Kafr El-Sheikh 08)*



8 **In the opposite figure :**

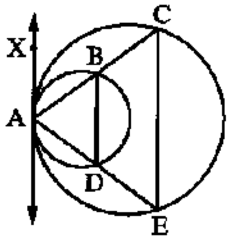
Two circles are touching internally at A

, $\overleftrightarrow{AX}$ is the common tangent to them at A

, $\overleftrightarrow{AB}$ and $\overleftrightarrow{AD}$ intersect the small circle at B , D

and the great circle at C , E

Prove that : $\overline{DB}$ // $\overline{EC}$ *(El-Gharbia 15 , El-Monofia 14 , Souhag 13)*



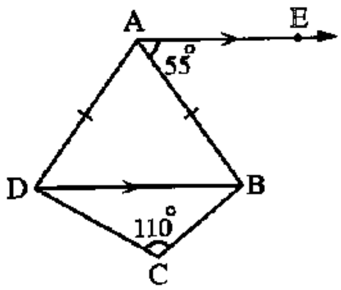
9 **In the opposite figure :**

$\overrightarrow{AE}$ // $\overline{DB}$, m ($\angle$ BAE) = 55° ,

m ($\angle$ C) = 110° and AB = AD

Prove that : (1) The figure ABCD is a cyclic quadrilateral.

(2) $\overrightarrow{AE}$ is a tangent to the circumcircle of the quadrilateral ABCD *(Beheira 05)*

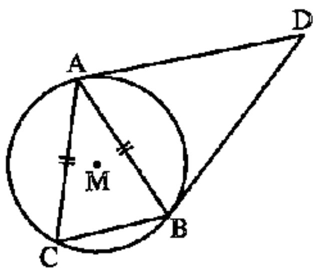


10 **In the opposite figure :**

$\overline{DA}$ and $\overline{DB}$ are two tangent-segments to the circle M at A and B

C $\in$ the circle M such that AB = AC

Prove that : $\overleftrightarrow{AC}$ is a tangent to the circumcircle of Δ ABD



Sheet (15)

Accumulative Basic Skills

Choose the correct answer :

1 The corresponding angles of the two similar polygons are .... in measure.

(a) equal (b) different (c) proportional (d) alternate

2 The area of a rhombus which the lengths of its diagonals are 6 cm. and 8 cm. equals ....

(a) 2 cm^2 (b) 14 cm^2 (c) 24 cm^2 (d) 48 cm^2

3 The number of axes of symmetry of two congruent circles and touching externally equals .... *(El-Dakahlia 16)*

(a) 4 (b) 2 (c) 1 (d) an infinite number.

4 The medians of a triangle meet at the same point which divides each in the ratio .... : .... from the base. *(Beni Suef 17)*

(a) 1 : 2 (b) 2 : 1 (c) 1 : 3 (d) 3 : 2

5 If the projection of a line segment on a straight line is a point , then the line segment .... the straight line. *(North Sinai 17 , Alexandria 16)*

(a) // (b) $\perp$ (c) $\in$ (d) $\subset$

6 ABC is a right-angled triangle at B where AB = 6 cm. , BC = 8 cm. , then its area = ... cm^2. *(Damietta 16)*

(a) 48 (b) 14 (c) 24 (d) 7

7 The length of the side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse. *(Ismailia 16)*

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{2}$ (d) 2

8 If m_1 and m_2 are two slopes of two parallel straight lines , then .... *(El-Fayoum 16)*

(a) $m_1 + m_2 = 0$ (b) $m_1 = m_2$ (c) $m_1 \times m_2 = -1$ (d) $m_1 - m_2 = -1$

9 The image of the point (2 , 3) by rotation R (O , 180°) is the point .... *(South Sinai 17)*

(a) (2 , 3) (b) (– 2 , 3) (c) (2 , – 3) (d) (– 2 , – 3)

10 If the side length of a rhombus is L cm. , then its perimeter = ... cm. *(New Valley 17)*

(a) L^2 (b) $2L^2$ (c) $4L$ (d) $2\sqrt{2}L$

11 The measure of the interior angle of the regular hexagon = ... *(Alex. 17)*

(a) $60°$ (b) $108°$ (c) $120°$ (d) $135°$

12 If M is a circle of radius length r cm. , then the length of the semicircle = ... cm.

(a) $2\pi r$ (b) $\frac{1}{4}\pi r$ (c) $\frac{1}{2}\pi r$ (d) πr

13 A square of perimeter 20 cm. , then its area = ... cm^2. *(Beni Suef 16)*

(a) 20 (b) 25 (c) 50 (d) 100

14 The two diagonals are equal in length and not perpendicular in the .... *(El-Menia 16)*

(a) square. (b) rhombus. (c) rectangle. (d) parallelogram.

15 If $\cos 2X = \frac{1}{2}$ where X is an acute angle , then $m(\angle X) =$ .... *(Beni Suef 16)*

(a) $15°$ (b) $30°$ (c) $45°$ (d) $60°$

16 Δ ABC is a right-angled triangle at C , then the two angles A and B are .... *(El-Menia 17)*

(a) supplementary. (b) complementary.

(c) adjacent. (d) vertically opposite angles.

17 Two parallel lines to a third are .... *(Luxor 16)*

(a) perpendicular. (b) parallel.

(c) intersecting. (d) skew.

18 The radius length of the circle whose centre is (7 , 4) and passes through the point (3 , 1) equals .... length units. *(Aswan 16)*

(a) 3 (b) 4 (c) 5 (d) 6

19 The number of symmetry axes of the square is .... *(El-Fayoum 17)*

(a) 1 (b) 2 (c) 3 (d) 4

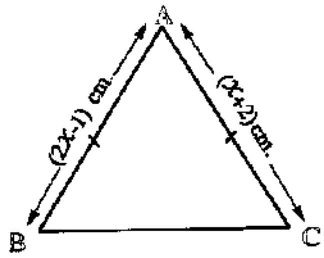
20 The numbers 5 , 4 and .... can be side lengths of a triangle. *(El-Menia 16)*

(a) 8 (b) 9 (c) 10 (d) 12

21 Δ XYZ is a right-angled triangle at Y , then XZ .... YZ *(North Sinai 17)*

(a) < (b) > (c) = (d) is twice

22 **In the opposite figure :**



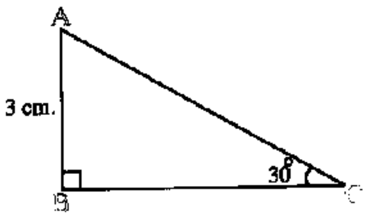
AB = AC , AB = $(2X - 1)$ cm. and AC = $(X + 2)$ cm. ,

then X = ... *(Cairo 16)*

(a) 3 (b) 5

(c) 11 (d) 14

23 **In the opposite figure :**



ABC is a right-angled triangle at B ,

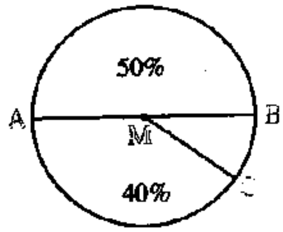
m ($\angle$ C) = 30° , AB = 3 cm. ,

then AC = ... cm. *(El-Fayoum 16)*

(a) 2 (b) 3

(c) $3\sqrt{3}$ (d) 6

24 **In the opposite figure :**



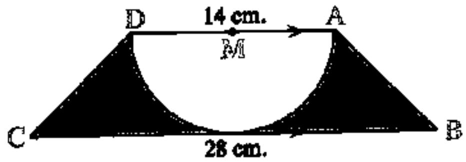
M is the centre of the circle ,

then m ($\angle$ CMB) = ... *(South Sinai 16)*

(a) 36° (b) 72°

(c) 144° (d) 180°

25 **In the opposite figure :**



ABCD is a trapezium in which $\overline{AD}$ // $\overline{BC}$

and $\overline{AD}$ is a diameter of circle M ,

then the area of the shaded region = ... *(Damietta 16)*

(a) 70 cm^2 (b) 147 cm^2

(c) 170 cm^2 (d) 224 cm^2

Best wishes